Orbit Spaces Arising from Isometric Actions on Hyperbolic Spaces

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Abstract

Let $G\times M\to M$ be a differentiable action of a Lie group G on a differentiable manifold M and consider the orbit space $\frac{M}{G}$ with the quotient topology. Dimension of $\frac{M}{G}$ is called the cohomogeneity of the action of G on M. If M is a differentiable manifold of cohomogeneity one under the action of G a compact and connected Lie group, then the orbit space $\frac{M}{G}$ is homeomorphic to one of the spaces $[0,1]$, $(0,1]$, S^1 or R. In this paper we suppose that the hyperbolic space H^n is of cohomogeneity two under the action of G, a connected and closed subgroup of $Iso(H^n)$. Then we prove that its orbit space is homeomorphic to R^2 or $[0,\infty)\times R$. Also we prove that either all orbits are diffeomorphic to R^{n-2} or there are nonnegative integers $m,\ e$ such that some orbits are diffeomorphic to R^e, and the other orbits are diffeomorphic to $B\times R^e$, where B may be a sphere, a homogeneous hypersurface of sphere or a helix in some Euclidean space.

Keywords: Manifold, Hyperbolic space, Cohomogeneity, Orbit space, Isometry

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