Normal forms of Hopf Singularities: Focus Values Along with some Applications in Physics

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Extended Abstract

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Introduction

This paper aims to introduce the original ideas of normal form theory and bifurcation analysis and control of small amplitude limit cycles in a non-technical terms so that it would be comprehensible to wide ranges of Persian speaking engineers and physicists. The history of normal form goes back to more than one hundreds ago, that is to the original ideas coming from Henry Poincare. This tool plays an important role in the bifurcation analysis of dynamical systems. Many phenomena in chemistry, physics, and engineering are modeled by parametric nonlinear differential systems. These systems demonstrate a complicated dynamics, when the parameters reach singular values. Normal form theory is an efficient tool for the local bifurcation analysis of singular nonlinear systems. The main idea relies in using a nonlinear change of coordinates to convert a given vector field into a simple form, which is named normal form.

In most nonlinear systems, behavior and dynamics is involved with local appearance or disappearance of small amplitude limit cycles. These are called bifurcations of limit cycles and they mostly occur through Hopf or degenerate Hopf bifurcations. Center manifolds and normal form theory are considered as among the most powerful tools for the bifurcations and stability analysis of limit cycles. Indeed, the analysis requires the computation of Lyapunov coefficients. Here we present the first (well-known) and the second Lyapunov coefficients for Hopf and degenerate bifurcations.

Lyapunov coefficients

In this paper, we discuss normal form of systems whose linear part has a pair of imaginary eigenvalues (usually referred by Hopf singularity) and compute the first and second Lyapunov coefficients.

In order to use the bifurcation analysis of a Hopf singularity for bifurcation control purposes, one needs to be able to derive the parametric normal forms of the original system so that the bifurcation analysis for the parametric normal form system would give rise to a bifurcation analysis in terms of the parameters of the original parametric system. We have designed a Maple program for this purpose. Here we merely present the Lyapunov coefficients. The program computes Lyapunov coefficients for systems with parameters and symbolic coefficients.

Application

Computation of Lyapunov coefficients is not important only in theatrical aspects but also in real world applications. So in last section of the Persian language paper, we apply the method on a well-known system, called Linard system, and a non-linear electrical system in order to illustrate how these tools can be applied in applications.

We detect Hopf singularity for these two application models and then compute their normal form up to degree 3.

Keywords: Normal Form, Hopf singularity, focus value, Linard system, Non-linear electrical system.

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