Universal Central Extension of the Tensor Algebra of a Lie Superalgebra and a Commutative Associative Algebra

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Introduction

Representation as well as central extension are two of the most important concepts in the theory of Lie (super)algebras. Apart from the interest of mathematicians, the attention of physicist are also drawn to these two subjects because of the significant amount of their applications in Physics. In fact, for physicists, the study of projective representations of Lie (super)algebras are very important.

Projective representations of a Lie superalgebra \mathfrak{L} are representations of the central extensions of \mathfrak{L} . So the study of projective representations has two steps; at first, one needs to know the central extensions and then to study their representations.

The first question in the study of central extensions is finding the universal one (if it exists). In 1984, universal central extensions of the algebras of the form $g = A \otimes \mathfrak{k}$, for a unital commutative associative algebra A and a simple finite dimensional Lie algebra \mathfrak{k} , were identified. Then in 2011, the case when \mathfrak{k} is a basic classical simple Lie superalgebra was studied by K. Iohara and Y. Koga. They first study the case for Lie superalgebras \mathfrak{k} of rank 1; then they study Z-forms of \mathfrak{k} and prove the existence of a Chevalley base type for \mathfrak{k} using its structure as a basic classical simple Lie superalgebra. This in particular helps them to define an even nondegenerate symmetric invariant bilinear form on \mathfrak{k} .

Material and methods

In this work, we study universal central extensions of Lie superalgebras of the form $A \otimes \mathfrak{k}$, where \mathfrak{k} is a finite dimensional perfect Lie superalgebra equipped with a nondegenerate homogeneous invariant supersymmetric bilinear form which is invariant under all derivations and A is a unital commutative associative algebra. Our techniques are totally different from the ones done before; in fact to get our results we use the materials of the previous work of the author (joint with Karl-Hermann Neeb) regarding central extensions of $A \otimes \mathfrak{k}$.

Results and discussion

We find the universal central extensions of Lie superalgebras of the form $A \otimes \mathfrak{k}$, where \mathfrak{k} is a finite dimensional perfect Lie superalgebra equipped with a nondegenerate homogeneous invariant supersymmetric bilinear form which is invariant under all derivations and A is a unital commutative associative algebra.

Conclusion

Universal central extensions of Lie superalgebras of the form $A \otimes t$ as above are identified. Our main result covers the results of the previous works in this regard and moreover, since odd nondegenerate invariant bilinear forms on t are allowed, we get something more, e.g., the uinversal central extension of A \otimes t for the queer Lie superalgebra t = q(n) is also covered by our main theorem.

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