Mixed Mode Oscillation in FitzHugh-Rinzel Model

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Extended Abstract

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Introduction

Since 1987 that J. Rinzel introduced his well-known model for elliptic bursters, which is today known as FitzHugh-Rinzel(FHR) model, a lot of work has been done on it and very interesting and complicated behaviors have been observed in this simple three-dimensional model. The FHR model is given by the following system of equations:

$$\begin{aligned} v' &= v - v^3 - w + y + I \\ w' &= \varphi(v + a - bw) \\ y' &= \varepsilon(-v + c - dy) \end{aligned} \tag{1}$$

It is a three-dimensional reduction of Hodgkin-Huxley equation introduced by Hodgkin and Huxley in 1952 for the modelling the propagation of an action potential in squid's axon. Rinzel reduces the dimension by introducing an algebraic relation between two channel variables in Hodgkin-Huxley model.

This simple model exhibits continuous spiking mode, bursting, chaotic and recently observed Mixed Mode Oscillation. Shilnikov et. al. discussed the transition between these regimes of behavior via reducing the system to equation-less, one-dimensional Poincare' mapping. By this method they investigated the transition between continuous spiking and bursting, bursting and mixed-mode oscillation, and mixed-mode oscillation and quiescence in FitzHugh-Rinzel model.

Usually authors take b in equation (1) as a parameter and discuss bifurcations that occur due to changes in this parameter. Also some others take c as a parameter. For instance, Shilnikov et. al. fixed a=0.7, b=0.8 and d=1. Then they let $\epsilon = 0.002$ and considered c as a bifurcation parameter (Shilnikov 2011). In this paper we consider one of the multiplier of first equation as a parameter and introduce some new kinds of MMO in FHR model.

Bursting is a behavior of the system that has an oscillating behavior at some interval of time and then returns to silence mode. This type of behavior can be repeated periodically. Bursting behaviors can have been observed in the voltage of neurons both in experiments and computational models.

MMO is a very interesting behavior of two time-scale dynamical systems, it alternates between two types of periodic behavior, large and small. This behavior can be periodic or aperiodic. MMOs have been observed in various systems such as chemical reactions (Petrov 1992), electrochemical systems (Koper 1991), neural systems (Rubin 2008), dusty plasma (Mikikian 2008) and climate model (Robert 2015). Desroches et al, have reviewed and classified the observed types of MMO in neural, physical and chemical systems (Desroches 2012 and references therein).

Desroches et al. also introduced some kind of signature for MMO that we use in this paper. For example this signature is denoted by $L_a^{s_b}$, that introduces the behavior specified by a large cycle and b small one. One can consider $L_1^{s_1} L_2^{s_2} L_3^{s_3} L_4^{s_4}$ ..., which refers to oscillating behavior that initiates with one big periodic orbits and pursues with one small ones, then goes one with two big periodic orbits and continues with two small ones and so on.

Material and methods

We consider the equivalent system to (1) introduced by Del Negro et al. which is given by the following system of equations:

$$\begin{aligned} v' &= 4v - 4v^3 - a * w - z + I \\ w' &= (1 + 4v + w) \\ y' &= \alpha(bv - (cz - z_0)/d) \end{aligned} \quad (2)$$

They take $a = 1$, $\alpha = 0.003$, b=1.25, c = 1, z_0=1.33 and d=4 but we consider the above system for different values of α, b, c, z_0 and d and we take a as a parameter. By considering the system as a fast-slow system and using geometric singular perturbation theory, we can find some interesting behaviors in the solutions of the above system. It is noticeable that the above model is a fast-slow system whose fast subsystem is a classical FitzHugh-Nagumo model which is a deep studied two dimensional model for neural behavior.

Results and discussion

Our model and the behavior of the fast subsystem has been analyzed. We depict the bifurcation diagram of the fast subsystem. Then we construct some types of MMOs and bursting solutions by using the information from the fast subsystem. For example we found $L_3^{s_1}$, $L_4^{s_1}$, $L_5^{s_1}$, $L_3^{s_1}L_4^{s_1}$, $L_2^{s_5}L_4^{s_2}$, $L_3^{s_3}$, $L_4^{s_4}$, $L_4^{s_1}L_4^{s_4}$, $L_9^{s_2}L_5^{s_1}$ and $L_4^{s_4}L_3^{s_2}$ by this technique. Also we found Fold-Hopf bursting, Big Homoclinic-Hopf bursting, SubHopf-SubHopf bursting and some Mixed type bursting according to Izhikevich classification of bursting solutions.

At the end we discussed the canard type MMO and study it carefully. Canard solutions have been observed in systems with two slow variables and one fast variable. The interesting point of the new canard type solution is that our system has one slow variable and two fast variables.

Keywords: FitzHugh-Nagumo Model, Mixed-Mode Oscillations, Bursting, Canard.

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