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Analytical Treatment of Volterra Integro-Differential Equations of Fractional Derivatives

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Abstract

In this paper the solution of the Volterra integro-differential equations of fractional order is presented. The proposed method consists in constructing the functional series, sum of which determines the function giving the solution of considered problem. We derive conditions under which the solution series, constructed by the method is convergent. Some examples are presented to verify convergence, efficiency and simplicity of the method.

Mathematics Subject Classification: 45J05, 65T60

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Introduction

In recent years, various kinds of analytical and numerical methods were used to solve fractional integro-differential equations. Rawashdeh [1] applied collocation method to study fractional integro-differential equations. Authors of [2] applied the Adomian decomposition method (ADM) to approximate solutions for fourth-order fractional integro-differential equations. The Haar wavelet method were used to solve the fractional integral equations [3]. Applied fractional differential transform method was employed to approximate solutions for fractional integro-differential equations in [4]. The homotopy analysis method for solving the fractional Volterra's population system has been presented in [5]. A recent application has included numerically determining solutions for various classes of nonlinear fractional equations in [6, 7, 8]. In this paper,

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we apply variational iteration method (VIM) [9, 10, 11, 12, 13, 14, 15] to solve Volterra integro-differential equations. This method is widely used by many researchers to study linear and nonlinear problems. This method is employed in [16] to solve the Klein-Gordon partial differential equations. Authors of [17] applied the variational iteration method to solve the Lane-Emden differential equation. For more applications of the method, we refere to [18, 19, 20, 21].

In this study, we consider Volterra integro-differential equations (VI-DEs) of fractional order of the form

$$\begin{cases} {}^{c}D_{0_+}^{\alpha}\zeta(x) - \lambda\int_0^x k(x,t)\zeta(\mathrm{t})\mathrm{dt} = \mathrm{g}(x)\ , \\ \zeta^{(\mathrm{i})}(0) = \mathrm{c_i}\,, \mathrm{i} = 0,1,2,\dots,\mathrm{n}-1\,, \mathrm{n} = [\alpha]+1\,. \end{cases} \tag{1.1}$$

where $g \in L^2([0,X]), k(\text{separable}) \in L^2([0,X]^2)$ are given functions, ${}^{c}D_{0_+}^{\alpha}$ is the fractional derivative, and ζ(x) is unknown function. The paper is organized as follows: In Section 2, we recall some basic definitions and properties of the fractional calculus theory. In Section 3, we construct an algorithm for solving Volterra integro-differential equations of fractional order by using the VIM. In Section 4, we derive the convergence conditions for the VIM to solve the Volterra integro-differential equations of fractional order. In Section 5, some illustrative examples are given. Some concluding remarks are given in Section 6.

Preliminaries

We now present the definitions and auxiliary results for fractional calculus which are used in this paper. For more details on the mathematical properties of fractional derivatives and integrals see ([22, 23]).

Definition 2.1. ([22]) The fractional integral of order $\alpha \geq 0$ of a function $\zeta(x) : (0,\infty) \to R$ is defined as

$$\begin{cases} I_{0_+}^{\alpha}\zeta(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-s)^{(\alpha-1)}\zeta(s)\mathrm{ds}\,, \\ I_{0_+}^{0}\zeta(x) = \zeta(x), \end{cases} \qquad \alpha > 0,$$

where Γ is the well-known Gamma function.

Definition 2.2 ([22]).The fractional derivative of order $\alpha \geq 0$ of a continuous function $\zeta(x)$: $(0,\infty) \to R$ is defined as

$${}^{c}D_{0_+}^{\alpha}\zeta(x) = \frac{1}{\Gamma(\mathrm{n}-\alpha)}\int_0^x (x-s)^{\mathrm{n}-\alpha-1}\,\zeta^{(\mathrm{n})}(s)\mathrm{ds}\,,\ \ \mathrm{n} = [\alpha]+\ 1\,.$$

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where [α] is the integer part of α.

Definition 2.3 ([23]). The Mittag-Leffler function $E_{\alpha,\beta}(z)$ with $\alpha > 0, \beta > 0$ is defined by the following series representation, which is valid in the whole complex plane

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha + \beta)} .$$

Lemma 2.1 ([22]). Riemann-Liouville factional integral and derivative have the following properties

$$(i) \quad I_{0_+}^{\alpha}\zeta(x)\ I_{0_+}^{\beta}\zeta(x) = I_{0_+}^{\alpha+\beta}\zeta(x) ,$$

$$(ii) \quad I_{0_+}^{\alpha}\zeta(x)\ I_{0_+}^{\beta}\zeta(x) = I_{0_+}^{\alpha}\zeta(x)\ I_{0_+}^{\beta}\zeta(x),$$

$$(iii) \quad I_{0_+}^{\alpha}x^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}x^{\alpha+\gamma}$$

$$(iv) \quad I_{0_+}^{\alpha}\ {}^{c}D_{0_+}^{\alpha}\zeta(x) = \zeta(x) - \sum_{k=0}^{n-1}\zeta^{(k)}(0^+)\frac{x^k}{k!} ,$$

$$(v) \quad {}^{c}D_{0_+}^{\alpha}\ I_{0_+}^{\alpha}\zeta(x) = \zeta(x) .$$

Main Results

Consider the following Volterra integral equation where ${}^{c}D_{0_+}^{\alpha}$ is the fractional derivative:

$$\begin{cases} {}^{c}D_{0_+}^{\alpha}\zeta(x) - \lambda\int_0^x k(x,t)\zeta(t)dt = g(x) , \\ \zeta^{(i)}(0) = c_i , i = 0,1,2,\dots,n-1 , n = [\alpha]+1 . \end{cases} \tag{3.1}$$

Theorem.3.1. The general solution of the Eq. (1.1) is equivalent to the mixed Volterra integral equation in the following form:

$$\zeta(x) = I_{0_+}^{\alpha}\big(g(x)\big) + \sum_{j=0}^{n-1} c_j\frac{x^j}{j!} + \lambda\frac{1}{\Gamma(\alpha)}\int_0^x\int_0^s (x-s)^{(\alpha-1)}\ k(x,t)\zeta(t)dtds.$$

Proof. Applying the operator $I_{0_+}^{\alpha}$, the inverse of the operator$D_{0_+}^{\alpha}$, to both sides of Eq. (3.1) yields:

$$\zeta(x) = I_{0_+}^{\alpha}\big(g(x)\big) + \sum_{j=0}^{n-1} c_j\frac{x^j}{j!} + \lambda\ I_{0_+}^{\alpha}\Big(\int_0^x k(x,t)\zeta(t)dt\Big)$$

which completes the proof.

3.1. Iterative method

Now we consider the **VI-DEs** as a mixed Volterra integral equation, which reads as:

$$\zeta(x) = I_{0_+}^{\alpha}\big(g(x)\big) + \sum_{j=0}^{n-1} c_j \frac{x^j}{j!} + \lambda\ I_{0_+}^{\alpha}\Big(\int_0^x k(x,t)\zeta(t)dt\Big). \tag{3.2}$$

There is a simple iteration formula for (3.2) in the form:

$$\zeta_k(x) = I_{0_+}^{\alpha}\big(g(x)\big) + \sum_{j=0}^{n-1} c_j \frac{x^j}{j!} + \lambda\ I_{0_+}^{\alpha}\Big(\int_0^x k(x,t)\zeta_{k-1}dt\Big),\ \ k = 1,2,\ldots. \tag{3.3}$$

Beginning with $\zeta_0(x) = I_{0_+}^{\alpha}\big(g(x)\big) + \sum_{j=0}^{n-1} c_j \frac{x^j}{j!}$, the approximate solution of Eq. (1.1) can be determined by the iterative formula Eq. (3.3).

3.2. Stability of the solution

We now investigate the changes in the obtained solution which are caused by small perturbations in the initial conditions.

Theorem.3.2 Let ζ(x) and $\tilde{\zeta}(x)$ be the solutions of the following problems:

$$\begin{cases} {}^cD_{0_+}^{\alpha}\zeta(x) - \lambda\int_0^x k(x,t)\zeta(t)dt = g(x)\,, \\ \zeta^{(i)}(0) = c_i\,, i = 0,1,2,\ldots,n-1, \qquad n-1 < \alpha \le n\,, \end{cases}$$

and

$$\begin{cases} {}^cD_{0_+}^{\alpha}\zeta(x) - \lambda\int_0^x k(x,t)\zeta(t)dt = g(x)\,, \\ \zeta^{(i)}(0) = b_i\,, i = 0,1,2,\ldots,n-1, \qquad n-1 < \alpha \le n\,, \end{cases}$$

where $|b_i - c_i| \le \epsilon_i, i = 0,1,2,\ldots,n-1$ and suppose that there exists a constant N such that:

$$|k(x,t)| \le N, \qquad \forall\ (x,t) \in [0,X]^2,$$

then we have

$$\left|\zeta(x) - \tilde{\zeta}(x)\right| \le \sum_{j=0}^{n-1} |\epsilon_j| x^j E_{\delta+1,j+1}(\ |\lambda|\ N\ x^{\delta+1})\,. \tag{3.4}$$

Proof. In accordance with the previous subsection, we have:

$$\zeta(x) = \lim_{k\to\infty} \zeta_k(x)\,,$$

$$\zeta_0(x) = I_{0_+}^{\alpha}\big(g(x)\big) + \sum_{j=0}^{n-1} c_j \frac{x^j}{j!}\,, \tag{3.5}$$

$$\zeta_k(x) = \zeta_0(x) + \lambda\ I_{0_+}^{\alpha}\Big(\int_0^x k(x,t)\zeta_{k-1}dt\Big),\ \ k = 1,2,\ldots. \tag{3.6}$$

and

$$\tilde{\zeta}(x) = \lim_{k\to\infty} \tilde{\zeta}_k\ ,$$

$$\tilde{\tilde{\zeta}}_0(x) = I_{0_+}^{\delta}\big(g(x)\big) + \sum_{j=0}^{n-1} b_j \frac{x^j}{j!}\,, \tag{3.8}$$

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$$\tilde{\zeta}_{\mathrm{k}}(x) = \tilde{\zeta}_0(x) + \lambda\ I_{0_+}^{\alpha}\left(\int_0^x k(x,t)\zeta_{\mathrm{k}-1}\mathrm{dt}\right), \quad \mathrm{k} = 1,2,\dots, \qquad (3.9)$$

From (3.5) and (3.8) it directly follows that:

$$\left|\zeta(x) - \tilde{\zeta}_0(x)\right| \leq \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right|\frac{x^{\mathrm{j}}}{\mathrm{j}!} \ . \qquad (3.10)$$

Using subsequently the relations (3.6) and (3.9), and the inequality (3.10), we obtain:

$$\begin{aligned}
\left|\zeta_1(x) - \tilde{\zeta}_1(x)\right| &\leq \left|\zeta_0 - \tilde{\zeta}_0\right| + |\lambda|\ I_{0_+}^{\alpha}\left(\int_0^x |k(x,t)|\left|\zeta_0 - \tilde{\zeta}_0\right|dt\right) \\
&\leq \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right|\frac{x^{\mathrm{j}}}{\mathrm{j}!} + \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right|\ |\lambda|\ \mathrm{N}\ \ I_{0_+}^{\alpha}\left(\frac{x^{j+1}}{(j+1)!}\right) \\
&= \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right|\left(\frac{x^{\mathrm{j}}}{\mathrm{j}!} + \mathrm{N}\ \frac{x^{j+1}}{\Gamma(j+\alpha+2)}\right) \\
&= \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right| \sum_{\mathrm{m}=0}^{1} |\lambda|^{\mathrm{m}}\ \mathrm{N}^{\mathrm{m}}\ \frac{x^{\mathrm{m}(\alpha+1)+\mathrm{j}}}{\Gamma(m(\alpha+1)+j+1)}.
\end{aligned}$$

Similarly, we have:

$$\begin{aligned}
\left|\zeta_2(x) - \tilde{\zeta}_2(x)\right| &\leq \left|\zeta_0 - \tilde{\zeta}_0\right| + |\lambda|\ I_{0_+}^{\alpha}\left(\int_0^x |k(x,t)|\left|\zeta_1 - \tilde{\zeta}_1\right|dt\right) \\
&\leq \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right| \sum_{\mathrm{m}=0}^{1} |\lambda|^{\mathrm{m}}\ \mathrm{N}^{\mathrm{m}}\ \frac{x^{\mathrm{m}(\alpha+1)+\mathrm{j}}}{\Gamma(m(\alpha+1)+j+1)},
\end{aligned}$$

and by induction:

$$\left|\zeta_k(x) - \tilde{\zeta}_k(x)\right| \leq \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right| \sum_{\mathrm{m}=0}^{1} |\lambda|^{\mathrm{m}}\ \mathrm{N}^{\mathrm{m}}\ \frac{x^{\mathrm{m}(\alpha+1)+\mathrm{j}}}{\Gamma(m(\alpha+1)+j+1)}. \qquad (3.11)$$

Taking the limit of (3.11) as $k \rightarrow \infty$, we obtain:

$$\begin{aligned}
\left|\zeta_k(x) - \tilde{\zeta}\ \ (x)\right| &\leq \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right| \sum_{\mathrm{m}=0}^{1} |\lambda|^{\mathrm{m}}\ \mathrm{N}^{\mathrm{m}}\ \frac{x^{\mathrm{m}(\alpha+1)+\mathrm{j}}}{\Gamma(m(\alpha+1)+j+1)} \\
&= \sum_{\mathrm{j}=0}^{\mathrm{n}-1}\left|\epsilon_{\mathrm{j}}\right|\ x^{\mathrm{j}}\ \mathrm{E}_{\alpha+1,\mathrm{j}+1}(|\lambda|\ \mathrm{N}\ x^{\alpha+1}),
\end{aligned}$$

which completes the proof.

It can be deduced from this theorem that small changes in initial conditions cause only small changes of the obtained solution.

Convergence Analysis

In order to prove that the sequence $\{\zeta_k(x)\}_{k=1}^{\infty}$ is convergent, we construct the series:

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$$\zeta_0(x) + [\zeta_1(x) - \zeta_1(x)] + \cdots + [\zeta_k(x) - \zeta_k(x)] + \ \ldots \ . \qquad (4.1)$$

Noticing that:

$$S_k(x) = \zeta_0(x) + [\zeta_1(x) - \zeta_0(x)] + \cdots + [\zeta_k(x) - \zeta_{k-1}(x)] = \zeta_k(x), \qquad (4.2)$$

So, the sequences $\{\zeta_k(x)\}_{k=1}^{\infty}$ will be convergent if all the series, $\sum S_k(x)$, are convergent.

Theorem 4.1 Consider the following equation:

$$\begin{cases} {}^{c}D_{0_+}^{\alpha}\zeta(x) - \lambda\int_0^x k(x,t)\zeta(\mathrm{t})\mathrm{dt} = \mathrm{g}(x) \ , \\ \zeta^{(\mathrm{i})}(0) = \mathrm{c_i}\,, \mathrm{i} = 0,1,2,\ldots,\mathrm{n}-1\,, \mathrm{n} = [\alpha]+1\,. \end{cases} \qquad (4.3)$$

The sequence defined by (3.3) with $\zeta_0(x) = I_{0_+}^{\alpha}\big(g(x)\big) + \sum_{\mathrm{j}=0}^{\mathrm{n}-1} \mathrm{c_j}\frac{x^{\mathrm{j}}}{\mathrm{j}!}$, converges to $\zeta(x)$ whenever $K1 = \|\zeta 0\ (x)\|_{\infty} < \infty$ and $K2 = \|k(x,t)\|_{\infty} < \infty$.

Proof. The assumptions imply the following estimations:

$$|\zeta_1(x) - \zeta_0(x)| \le |\lambda| \ I_{0_+}^{\alpha}(\int_0^x k(x,t)\left|\zeta_0(\mathrm{t})\right|\mathrm{dt})$$
$$\le |\lambda|\ \mathrm{K_1K_2}\ I_{0_+}^{\alpha}(\int_0^x \mathrm{dt}) = K_1|\lambda|\mathrm{K_2}\ \frac{x^{\alpha+1}}{\Gamma(\alpha+2)}.$$

Similarly, we have:

$$|\zeta_2(x) - \zeta_1(x)| \le |\lambda| \ I_{0_+}^{\alpha}(\int_0^x k(x,t)\left|\zeta_1(\mathrm{t}) - \zeta_0(\mathrm{t})\right|\mathrm{dt})$$
$$\le K_1|\lambda|^2 K_2^2 \ \frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)}.$$

Proceeding by induction we obtain:

$$|\zeta_k(x) - \zeta_{k-1}(x)| \le K_1|\lambda|^k K_2^k \ \frac{x^{k\alpha+1}}{\Gamma(k\alpha+2)}.$$

Since the series of:

$$\sum K_1|\lambda|^k K_2^k = K_1\, x\, E_{\alpha,2}\ \left(|\lambda|\ \mathrm{K_2}\ x^{\delta}\right),$$

is convergent in the whole real line, therefore the series of (4.1) is absolutely convergent, which means that the sequence $\{\zeta_k(x)\}_{k=1}^{\infty}$ is convergent for $x \in [0,X]$.

Test examples

In order to demonstrate the performance of the present method as a novel solver for integro-differential equations of fractional order, two different problems are selected as test cases. The calculations were done in MAPLE.

Example.5.1 Consider the following integro-differential equation with the following

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initial condition:

$$\begin{cases} {}^{c}D_{0_+}^{\alpha}\zeta(x) - \int_0^x (x-t)\zeta(\mathrm{t})\mathrm{dt} = x \ , 0 < \alpha \le 1 \, , \qquad 0 \le x \le 1 \, , \\ \zeta(0) = 0 \, . \end{cases} \tag{5.1}$$

Applying the operator $1O_{+}^{\alpha}$, the inverse of the operator ${}^{\mathrm{c}}D_{0_+}^{\alpha}$, to both sides of the above equation yields:

$$\zeta(x) - \ I_{0_+}^{\alpha}(\int_0^x (x-t)\zeta(\mathrm{t})\mathrm{dt}) = \frac{x^{\alpha+1}}{\Gamma(\alpha+2)}, 0 \le x \le 1.$$

Its iteration formula reads:

$$\zeta_{\mathrm{k}}(x) - \ I_{0_+}^{\alpha}(\int_0^x (x-t)\zeta_{\mathrm{k}-1}(\mathrm{t})\mathrm{dt} = \frac{x^{\alpha+1}}{\Gamma(\alpha+2)}, 0 \le x \le 1 \, ,$$

and

$$\zeta_0(x) = \ \frac{x^{\alpha+1}}{\Gamma(\alpha+2)}.$$

Therefore, we have:

$$\begin{cases} \zeta_1(x) = \dfrac{x^{\alpha+1}}{\Gamma(\alpha+2)} + \dfrac{x^{2\alpha+3}}{\Gamma(2\alpha+4)} \\ \zeta_2(x) = \dfrac{x^{\alpha+1}}{\Gamma(\alpha+2)} + \dfrac{x^{2\alpha+3}}{\Gamma(2\alpha+4)} + \ \dfrac{x^{3\alpha+5}}{\Gamma(3\alpha+6)} \\ \zeta_3(x) = \dfrac{x^{\alpha+1}}{\Gamma(\alpha+2)} + \dfrac{x^{2\alpha+3}}{\Gamma(2\alpha+4)} + \dfrac{x^{3\alpha+5}}{\Gamma(3\alpha+6)} + \dfrac{x^{4\alpha+7}}{\Gamma(4\alpha+8)}. \end{cases}$$

By continuing this way, we obtain:

$$\zeta_{\mathrm{k}}(x) = \sum_{\mathrm{j}=0}^{\mathrm{k}} \frac{x^{j(\alpha+2)+(\alpha+1)}}{\Gamma(\mathrm{j}(\alpha+2) + \ (\alpha+2))} \, , \tag{5.2}$$

and the exact solution is:

$$\lim_{k\to\infty} \zeta_{\mathrm{k}}(x) = \sum_{\mathrm{j}=0}^{\infty} \frac{x^{j(\alpha+2)+(\alpha+1)}}{\Gamma(\mathrm{j}(\alpha+2) + \ (\alpha+2))} \, . \tag{5.3}$$

It is clear that:

$$\sum_{\mathrm{j}=0}^{\infty} \frac{x^{j(\alpha+2)+(\alpha+1)}}{\Gamma\big(\mathrm{j}(\alpha+2) + \ (\alpha+2)\big)} = \ x^{\alpha+1}E_{\alpha+2,\alpha+2}\,(x^{\alpha+2}).$$

Thus the above sequence is convergent. This confirms that the VIM for integro-differential equation in (5.1) converges to the exact solution.

Example.5.2 Consider the following integro-differential equation with the following initial condition:

$$\begin{cases} {}^{c}D_{0_+}^{\alpha}\zeta(x) - \int_0^x (x\sin(t)\,\zeta(\mathrm{t})\mathrm{dt}) = \mathrm{g}(x)\ , 0 < \alpha \le 1\,, \\ \zeta(0) = 0\,. \end{cases} \tag{5.4}$$

where:

$$g(x) = \cos(x)(x^3 + x^2 - 2x) - x\sin(x) + 2x + \frac{2}{\Gamma\left(\frac{5}{2}\right)} x^{\frac{3}{2}} + \frac{1}{\Gamma\left(\frac{3}{2}\right)} x^{\frac{1}{2}}. \tag{5.5}$$

Applying the operator $I_{0_+}^{\frac{1}{2}}$, the inverse of the operator ${}^{c}D_{0_+}^{\frac{1}{2}}$, to both sides of the above equation yields:

$$\zeta(x) - I_{0_+}^{\frac{1}{2}}(\int_0^x x\sin(t)\,\zeta(\mathrm{t})\mathrm{dt}) = x^2 + x, 0 \le x \le 1.$$

Its iteration formula reads:

$$\zeta_{\mathrm{k}}(x) - I_{0_+}^{\frac{1}{2}}(\int_0^x x\sin(t)\,\zeta_{\mathrm{k}-1}(\mathrm{t})\mathrm{dt}) = x^2 + x, 0 \le x \le 1,$$

and

$$\zeta_0(x) = x^2 + x.$$

Then, we obtain $\zeta_1, \zeta_2, \ldots$ as

$$\zeta_k\,(x) = x^2 + x, k = 1, 2, 3, \ldots.$$

Hence $\zeta(x) = lim_{k\to\infty}\,\zeta_k\,(x) = x^2 + x$ is the exact solution of (5.4).

Conclusions

In this paper, we applied the VIM to integro-differential equations of fractional order. We have shown that some appropriate conditions guarantee the convergence of the approach. Some examples are presented to illustrate the accuracy of the present method. The numerical cases reveal that obtained solutions are in good agreement with the exact ones. The numerical results show that variational iteration method is very effective and convenient for solving integro-differential equations of fractional order.

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