A RBF partition of unity collocation method based on a finite difference scheme to solve parabolic stochastic partial differential equations

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Extended Abstract

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Introduction

The study of parabolic stochastic partial differential equations (PSPDEs) attracted a lot of interest due to the existence of different stochastic terms and uncertainty in many phenomena. This is the reason that PSPDEs are able to fully capture the behavior of such phenomena. However, it is difficult to obtain the analytical solutions of PSPDEs. Thus, the numerical solutions of PSPDEs become a fast-growing research area. Several competitive methods are discussed for approximating PSPDEs, including the finite difference (FD) method and the finite element (FE) method, but all these numerical methods are based on a grid discretization that has to improve the solution through adaptive meshing. The mesh generation is one of the biggest challenges in mesh-based methods. To overcome this difficulty, meshless methods are powerful numerical techniques that have been applied to solving PSPDEs. And consequently, the purpose of meshless methods is to eliminate the structure of the mesh. A class of meshless methods is based on radial basis function (RBF) methods which put radial functions as the basis functions for the collocation. The main advantages of these methods lie in their simplicity and their effectiveness in dealing with high-dimensional problems with complicated geometries since no mesh generation is needed.

However, the global RBF collocation methods have some disadvantages for the numerical solution of time-dependent PSPDEs. Estimates of condition numbers of the matrices in the resulting dense linear systems indicate that the meshless method using RBFs in global view may be unstable at each realization to solve PSPDEs. Also, the computational cost is the main barrier when using global RBF methods.

In order to avoid numerical instabilities in global RBF methods, we are interested in the use of local RBF methods for the numerical solution of PSPDEs. In this paper, the RBF partition of unity collocation method based on a FD scheme (RBF-PU-FD) as a localized RBF approximation presented to deal with these issues. Finally, in order to show the efficiency of the proposed approach, we compare the results with the global RBF-FD method.

Material and methods

In this scheme, FD approximation leads to time derivative discretization for solving time-dependent PSPDE, and finally, we can arrive at a system of equations according to the stepwise structure. For this purpose, we simulate the Gaussian random field with spatial covariance structure at a finite collection of predetermined collocation points. We then apply localized RBF approximation using PU structure on the spatial discretization.

Results and discussion

We focus on a wide series of experiments, which concern the 2D and 3D stochastic heat problems. The gained results show the capabilities and efficiency of the RBF-PU-FD method for time-dependent PSPDEs with high dimensional and complicated geometries. Also, our method gives statistical criterions such as mean, standard deviation, lower bound and upper bound for prediction which are evaluated using the Monte-Carlo method.

Conclusion

The following conclusions were drawn from this research.

- The RBF-PU-FD collocation method allows overcoming the high computational cost associated with the ill-conditioned and dense matrices of global RBF method, while maintaining high accuracy, since the matrices formed during the RBF-PU-FD method are sparse.
- The convergence properties of the local approximations can be leveraged, while local couplings between approximations on different patches are enforced through the PU framework.
- The advantage of this approach is that a large problem is decomposed into many small problems and therefore, in the approximation procedure we can also work with a large number of nodes.
- RBF-PU-FD method is a simple and computationally efficient tool in dealing with high-dimensional PSPDE with complicated geometries.
- The RBF-PU-FD technique preserves the local approximation order for the global fit, while at the same time ensuring that the accuracy obtained for the local fits is carried over to the global one.

Keywords: Parabolic stochastic partial differential equations, Meshless methods, RBF collocation, Partition of unity method.

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