ON DENSE LEFT IDEAL PURE *S*-ACT

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Received: 04/16/2018 Accepted: 12/29/2019

Extended Abstract

Paper pages (495-512)

Introduction

The notion of purity in the sense of solubility of the system of equations is an important and interesting subject in different branches of mathematics and has been studied in various fields of mathematics since 1955. This concept can be generalized by considering a specific set of equations. Using the concept of torsion theory, Lembek defines a special kind of pure submodules in the category of R-Mod. Here, similar to the Lambek's idea, we introduce a certain kind of pure subacts associated with each Hoehnke radical, using the existence of one-one correspondence between torsion theories and Kurosh-Amitsur radicals in the category t of S-acts, and study the related notions.

Material and methods

Given a Hoehnke radical r in the category **S-Act**, a subact B of an S-act A is called *dense left ideal pure* or simply *d.l.i.pure* whenever Da ≤ B, for some a ∈ A and some r-dense left ideal D of S, implies that there exists b ∈ B such that da = db, for every d ∈ D, meaning that every system of equations of the form $\{da = dx\}_{d\in D}$has a solution in B. In this paper, we study this kind of purity and characterize the d.l.i-pure subacts, as well as some of its features.We also describe monoids whose actions are absolutely d.l.i.pure.

Results and discussion

Considering the d.l.i-pure subacts of an S-act, we give several characterizations of absolutely d.l.i.pure S-acts by some related notions. In particular, we show that the notions of absolutely d.l.i.purity and r-weakly injectivity coincide, for every Hoehnker adicalr. We eventually correlate a closure operator, c_D with each r-dense left ideal D, and use it to characterize the totally d.l.i.pure S-acts.

Conclusion

The following conclusions were drawn from this research:

● Let B be a subact of an S-act A. Then every system $\Sigma = \{s_i b_i = b_i \mid s_i \in S, b_i \in B, and\ i \in\}$ which is soluble in A has a solution in B if and only if B is left ideal pure in A.

● Given a Hoehnke radical r, every r-closed subact B of an S-act A is d.l.i.pure in A.

● The following conditions are equivalent.

(1) *D is an r-dense left ideal of S.*

(2) *Da = {b} implies that Sa = {b} and b is a zero element, for every A ∈ Sr and a,b∈A.*

(3) *Da = {b} implies that a = b and b is a zero element, for every A ∈ Sr and a,b ∈ A.*

● The following conditions are equivalent, for an S-act A:

(4) *A is absolutely d.l.i.pure.*

(5) *A is d.l.i.pure in each r-injective extension of it.*

(6) *A is d.l.i.pure in its r-injective hull, $E_r(A)$).*

(7) *A is a weakly r-injective S-act.*

(8) *A is d.l.i.pure in an r-injective extension I of A.*

● Every d.l.i.pure subact of an absolutely d.l.i.pure S-act is an absolutely d.l.i.pure S-act.

● Every r-closed subact of an absolutely d.l.i.pure S-act is absolutely d.l.i.pure.

● The following conditions are equivalent:

(1) The monoid S is completely d.l.i.pure, i.e. every S-act is absolutely d.l.i.pure.

(2) Every S-act is d.l.i.pure in each r-injective extension of it.

(3) Every S-act is d.l.i.pure in its r-injective hull.

(4) Every S-act is weakly r-injective.

(5) Every S-act is d.l.i.pure in an r-injective extension of itself.

(6) Every r-dense left ideal D is d.l.i.pure in S.

(7) Every r-dense left ideal D of S has a right identity e_D.

(8) Every r-dense left ideal of S is generated by an idempotent element.

● Given an r-dense left ideal D of S, the family $c = (c_A^D)_{A\in S-Act}$ defined by c_A^D(B)= <{a ∈ A | Da ≤ B}>, for every subact B of an S-act A, is a closure operator.

● Let B be a radical subact of an S-act A. Then c_A^D(B) is a radical subact of A, for every r-dense left ideal D of S.

● Let B be a d.l.i.pure subact of an S-act A. Then B is r-dense in C in c_A^D(B), for every r-dense left ideal D of S.

Keywords: Radical, S-act, r-dense, d.l.i.pure.

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