On Concircular transformations in Finsler spaces

Samaneh Sadat Saberali^{*}, Bahman Rezaei;

Faculty of Mathematical Sciences and Statistics, Urmia University, Urmia, Iran

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Extended Abstract

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Introduction

A geodesic circle in an Euclidean space is a straight line or a circle with finite positive radius, A geodesic circle in a Riemannian geometry, as well as in Finsler geometry, is a curve for which the first Frenet curvature k_1 is constant and the second curvature k_2 vanishes. In the other words a geodesic circle is a torsion free constant curvature curve.

A geodesic circle can be generalized naturally to Riemann geometry by using Levi-Civita connection, or more generally generalized to Finsler geometry by the so called Cartan Y –connection introduced by M. Matsumoto.

In 1940s, Yano introduced concircular transformations of Riemannian manifolds and developed the theory of concircular geometry in a series of papers, After that, some researchers did further jobs on concircular transformations in Riemann geometry. Vogel shows that a concircular transformation of Riemannian manifolds is a conformal transformation and Ishihara proves that a concircular vectorfield on a Riemannian manifold is a conformal vector field.

Material and methods

In this paper, first we consider conformal transformations and with using a special class of Finsler metrics that are Randers metrics find some new results that can be expressed in this paper, More precisely we studied conformally related Randers metrics then by using some curvatures for example weak isotropic flag curvature and weak Berwald curvature also Einstein metrics which can be solved via a suitable methods.

Results and discussion

We solve some theorems by using present techniques to demonstrate the efficiency, high accuracy and the simplicity of the present method, then we get new results in concircular transformations. The reported results demonstrate that there is a good agreement between conformal transformations and concircular transformations. Notice that, obtained results confirm that proposed method enables us to find some more reasonable results in circle preserving transformations.

Conclusion

The following conclusions were drawn from this research.

- Conformally related Randers metrics via Weak Einstein metrics that related with concircular transformations are found very easily and therefore many calculations are reduced.
- Conformally related Randers metrics via weak isotropic flag curvature is considered, so proposed method is easy to implement and it is suitable to obtain solutions of various kind of problems with little additional works.
- Randers metrics with conformal transformations by using weakly mean Berwald curvature which can be easily solved by using direct method.

Keywords: Concircular transformation, Finsler metric, Isotropic flag curvature, mean Berwarld curvature

*Corresponding author: s.saberali@urmia.ac.ir