Semi Armendariz and Semi McCoy rings

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Extended Abstract

Introduction

Liu and Zhao called a ring R weak Armendariz if whenever polynomials $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{j=0}^{m} b_j x^j$ in $R[x]$ satisfy $f(x)\, g(x) = 0$, then for all i and j, $a_i b_j \in Nil(R)$ (the set of nilpotent elements of R). In 2017 Sannaei, Sahebi and Javadi called a ring R, J-Armendariz if whenever polynomials $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{j=0}^{m} b_j x^j$ in $R[x]$ satisfy $f(x)g(x) = 0$, then for all i and j, $a_i b_j \in J(R)$(the radical Jacobson of R).Victor Camillo, Tai Keun Kwak, and YangLee called a ring R right NC-McCoy if whenever polynomials $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{j=0}^{m} b_j x^j$ in $R[x]$ satisfy $f(x)\, g(x) = 0$, there exists a nonzero element r in R such that $f(x)r \in Nil(R)$. Left NC-McCoy rings are defined analogously, and a ring R is called NC-McCoy if it is both left and right NC-McCoy. In 2016 Vahdani, Sahebi and Javadi called a ring R, right J-McCoy if whenever polynomials $f(x)$ and $g(x)$ in $R[x]$ satisfy $f(x)\, g(x) = 0$, then there exists nonzero element c in R such that $a_i r \in J(R)$for all i. Left J-McCoy rings are defined analogously, and a ring R is called J-McCoy if it is both left and right J-McCoy.Clearly, weak Armendariz (resp. NC-McCoy) rings are J-Armendariz (resp. J-McCoy) and for an artinian ring, weak Armendariz rings (resp. NC-McCoy) andJ-Armendariz (resp. J-McCoy) rings are the same.

Materials and methods

In this paper, we investigate about a subclass of J-Armendariz (resp. J-McCoy) rings. We call a ring R, Semi Armendariz (resp. Semi McCoy) if $\overline{R} = \frac{R}{J(R)}$ is Armendariz (resp. McCoy). we study Semi Armendariz (resp. Semi McCoy) property with respect to some standard constructions like direct product, factor rings, subrings, matrix rings, corner rings, polynomial rings, etc.

Results and discussion

We show that the class of Semi Armendariz (resp. Sem iMcCoy) rings lies properly between the class of one-sided quasi-duo rings and the class of J-Armendariz (resp. J-McCoy) rings.We prove that a ring R is Semi Armendariz (resp. Semi McCoy) iff $R[[x]]$ isSemi Armendariz (resp. Semi McCoy) iff for any idempotent $e \in R$, eRe isSemi Armendariz (resp. Semi McCoy) iff the n-by-n upper triangular matrix ring $T_n(R)$is Semi Armendariz (resp. Semi McCoy).

Conclusion

The following conclusions were drawn from this research.

- Semi Armendariz (resp. Semi McCoy) property is not Morita Invariant.
- Polynomial rings on Semi Armendariz rings are not necessarily Semi Armendariz.
- Subrings of Semi Armendariz rings are not necessarily Semi Armendariz.
- Although, Armendariz rings are abelian but Semi Armendariz rings are not necessarily abelian.

Keywords: J-Armendariz ring, J-McCoy ring, Quasi duo-ring, Jacobson radical.

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