

The uniqueness of solution for boundary value problems including elliptic Cauchy-Riemann equation with local and nonlocal boundary conditions

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Extended Abstract

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Introduction

Cauchy-Riemann equation has an important role in the theory of complex analysis in mathematics. Sometimes, by making use of this equation, the possibility of continuing of an analytic function is discussed.

On the other hand, this equation appears in some of boundary value problems in mathematical physics. Existence and uniqueness of solution for these boundary value problems are very important. Several methods have been applied for the existence and uniqueness of The solution.

Moreover, this equation has a strongly relation with Laplace equation. The solution of Laplace equation are harmonic functions. The Laplace equation can decompose to the Cauchy- Riemann equations. There are an affective relation between the fundamental solutions of Laplace equation and Cauchy- Riemann equations.

Material and methods

In this paper, we will use a new method to establish the uniqueness of solution for a boundary value problem including Cauchy-Riemann equation. This method is based on Astrogradsky formula and some necessary conditions which are obtained by this formula and some calculations in boundary integral equations and the property of Delta Dirac function. Then, the kernel of boundary integral equations are regularized by a special method which devoted to the authors.

Finally, the regularized kernels are estimated by some of mathematical analysis theorems such as mean value theorem, to use the contraction map theorem for proving the uniqueness of solution.

Results and discussion

In this paper, two boundary value problems including Cauchy- Riemann equation with two different boundary conditions are considered. In first case, the boundary condition is local, and for proving the uniqueness of solution, we show that the related homogenous problem has only trivial solution.

In second case, the boundary condition is non-local, and for establishing the uniqueness of solution, at first, the given boundary value problem is reduced to the second kind of Fredholm boundary integral equations with regularized kernels. Then some sufficiently conditions are given such that the regularized kernels satisfy in contraction mapping theorem conditions to establish the uniqueness of solution.

Conclusion

The following conclusions were drawn from this research.

- In the first section of this paper, the boundary value problem is given with local boundary conditions. And the uniqueness of solution is proven via the related homogenous problem has only trivial solution.
- But, in the second section of this paper, the boundary value problem is given with nonlocal condition.
- The uniqueness of this problem is proven via different method, by making use of contraction mapping theorem.

Keywords: Boundary value problem, Cauchy- Riemann equation, Boundary integral equations, Contraction mapping theorem.

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