

On the Classifying of the Tangent Sphere Bundle with Almost Contact B-Metric Structure

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Received: 08/04/219 Accepted: 12/17/2019

Extended Abstract

Paper pages (467-484)

Introduction

One of the classical fundamental motifs in differential geometry of manifolds is the notion of the almost contact structure. As a counterpart of the almost contact metric structure, the notion of the almost contact B-metric structure has been an interesting research field for many mathematicians in differential geometry of manifolds, and the geometry of such structures has been studied frequently. There is a classification for the almost contact B-metric structures, named the relevant classification, with respect to the covariant derivative of the fundamental tensor of type $(1, 1)$. In this paper, we basically use this classification to achieve our goals. On the other hand, many of mathematicians have widely considered the concept of lifted metric on the tangent bundle and tangent sphere bundle of a Riemannian manifold (M, g) . The idea of constructing a lifted metric on the tangent bundle was a strong inspiration for many of mathematicians and finally, the notion of g -natural metric as the most general type of lifted metrics on tangent bundle TM of a Riemannian manifold (M, g) was introduced in 2005. In this paper, we consider a pair of associated g -natural metrics on the unit tangent sphere bundle T_1M with B-metric, and we classify this structure with respect to the relevant classification of almost contact manifold with B-metric.

Material and methods

In this paper, first we introduce the notion of natural metrics on the unit tangent sphere bundle and also, we present an almost contact B-metric structure on T_1M . Then, by using the structural tensor F , we investigate the conditions of each eleven classes of the relevant classification of the almost contact B-metric manifolds.

Results and discussion

We consider the unit tangent sphere bundle of a Riemannian manifold (M, g) as a $(2n+1)$ dimensional manifold, and then we equip that to the g -natural metric. Considering the associated metric of this metric, we define an almost contact B-metric structure on the unit tangent sphere bundle, and by using the

coefficients of the structural tensors and the relevant classification for such structures, we attempt to classify the unit tangent sphere bundle equipped to this structure, and we prove that the non-zero coefficients of structure tensor F belong to the class $F_1 \oplus F_4$.

Conclusion

The following conclusions are obtained from this research.

- The two non-zero coefficients of structure tensor F (under some curvature conditions) belong to $F_1 \oplus F_4$, which we denote it by U_1 .
- All the zero coefficients of structure tensor F satisfy classes F_5 and F_{10} and F_{11} .
- If M is a flat Riemannian manifold and T_1M is the unit tangent sphere bundle, then T_1M belongs to the class F_0 .

Keywords: Almost contact structure, B-metrics, Natural metric, Sphere bundle, Structure tensor.

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