Expected Bayesian Estimator and Hierarchical Bayesian Estimator for the Parameter of a Rayleigh Distribution Reliability System Under the Progressive Type-II Data Sample

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Received: 11/09/2019 Accepted: 06/14/2018

Extended Abstract

Paper pages (527-544)

Introduction

Rayleigh distribution is one of the most widely used distributions in reliability. For the first time, Lord Rayleigh (BalaKrishnan and Johnson (1994), Continuous Variable Distributions) introduced the Rayleigh probability distribution and examined the properties of this distribution. Rayleigh distribution is widely used in statistical modeling, it also plays an important role in vacuum electrical devices and communication engineering. Rayleigh probability distribution is usually used in cases where the variable under study has two independent components so that both have a normal distribution with the same variance. The Rayleigh probability density function of the Rayleigh distribution with the scale parameter θ is defined as follows.

$$f(x|\theta) = 2\theta x e^{-\theta x^2} , \qquad x, \theta > 0 \tag{1}$$

Also, the Rayleigh probability distribution function is as follows.

$$F(x|\theta) = 1 - e^{-\theta x^2}, \qquad x, \theta > 0 \tag{2}$$

The idea of prior hierarchical distribution was first proposed by Lindley and Smith (1972), and then Han (1997) generalized the structure of previous hierarchical distributions and introduced its applications. In this regard, several articles were published. For further study and observation of applications of the hierarchical Bayesian method, we can refer to Han (1998) to estimate the failure rate in exponential distribution, Ling and Shi (1999) to estimate the parameters of Pascal, Han and Lee (1999) used to estimate system reliability and Han (1999) used to estimate failure rate.

In the hierarchical approach proposed by Lindley and Smith (1972), the super parameter in the previous distribution of θ is considered as a random variable with a specific distribution. In this method, suppose b is a superparameter of the distribution parameters θ. Suppose θ has an prior density function $\pi_2(b)$ and the parameter cloud b has an prior density function $\pi_2(b)$. In this case, the hierarchical density function θ is defined as follows.

$$\pi(\theta) = \int \pi_1(\theta|b)\pi_2(b)db \ , \qquad b \in \Lambda \tag{3}$$

Finally, Han (2004) generalized this method and introduced a new idea called the expected Bayesian estimators. In recent years, many authors have presented articles on the theoretical and practical aspects of hierarchical and expected Bayesian estimators. For further study of the expected Bayesian approachcan refer to Han & Ding (2004) can be used to estimate failure probability parameters and failure rate, Han (2005) can be used to estimate reliability parameters, and Han (2007) can be used to estimate failure probability.

Material and Methods

In lifetime and reliability tests, the units that are tested are sometimes lost or decommissioned before failure, such units are usually called censored units. To be. The sampling scheme of the second type of progressive censored is as follows: Suppose in a lifetime test, n units are tested at zero time and we want m to be observed until failure (death). When the first failure is observed (i.e. the time of observation of the first sample unit $x_{r_1:m,n}$), r_1 is randomly selected from the remaining units and removed from the experiment. At the time of observation of the second failure (i.e. the time of observation of the second sample unit $x_{r_2:m,n}$), r_2 of $n - r_1 - 1$ the remaining units are randomly selected and tested. Finally, at the time of observation of the failure of m (ie the time of observation of the mth of the sample unit $x_{r_m:m,n}$), all remaining units, i.e. $r_m = n - m - r_1 - r_2 - \cdots - r_{m-1}$ up to the test units Are deleted. In this censorship scheme, all censorship values, i.e.$r_1, r_2, \dots, r_m$, are all predefined.

Results and discussion

Expected Bayesian estimator: In calculating the Bayesian estimator θ, the hyperparameters contained in it are assumed to be unknown but constant variables. But in the hierarchical approach of Lindley and Smith (1972), these parameters appear as random variables in the model to have a more reasonable posterior density function.

Hierarchical Bayesian estimator: If b is a parametric cloud in parameter θ and the random variable θ has an prior density function $\pi_1(\theta|b)$ and the prior density function of parameter cloud b is π_2(b), then according to Lindley and Smith (1972) the density function The previous hierarchy corresponding to θ is equal to

$$\pi(\theta) = \int_{\Lambda} \pi_1(\theta|b)\pi_2(b)db \ , \qquad b \in \Lambda$$

Conclusion

Our main purpose in this paper was to investigate the relationship between expected Bayesian estimators and hierarchical Bayesian estimators.We obtained the second, then by considering three different distributions for the hyperparameters that appeared in the previous distribution, the hierarchical Bayesian estimators and the expected Bayesian estimators are presented in theorems (1) and (2), respectively, and finally to investigate. The relationship between the obtained estimators of Theorem (3) was presented.

Keywords: Expected Bayesian estimator, Rayleigh distribution, Prior distribution, Hierarchical distribution.

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