Numerical solution of fractional delay Volterra integro-ifferential equations Using Shifted Orthogonal Genocci Polynomials

Leila Mansouri[1], Esmail Babolian[*2];
1. Department of Mathematics, College of Science,Yadegare-e-Imam Khomeini (RAH), Iran.
2. Faculty of Mathematical Sciences and Computer, Kharazmi University, Tehran, Iran.

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Extended Abstract

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Introduction

Recently, various authors have studied fractional calculus and its applications in earthquake, solid mechanics, dynamic fluid traffic, statistical mechanics, viscoelastic materials, economics, transportation, bioengineering, etc. These problems usually do not have analytic solution. Consequently, different numerical methods have been developed to approximate these equations. Also, fractional delay differential equations were considered in various papers. In the present paper, we are going to solve the equation

$$\begin{cases} D^{\beta}u(t) = F\left(t, u(t), u(\,b(t,u(t)), \int_{t-h}^{t} g\big(t,s,u(s)\big)\,ds\right), & t \geq ۰, \\ u^{(i)}(۰) = u_{۰i}, \qquad i = ۰, ۱, \dots, \mathscr{g} - ۱, & \mathscr{g} = \lceil \beta \rceil. \end{cases} \tag{۱}$$

The main goal is to approximate the solution of equation (1) by Galerkin method with shifted orthogonal Genocci polynomials and study the error of the method.

Material and methods

In this scheme, we first obtain the operational matrix for fractional derivatives in the Caputo sense and delayed operational matrices based on shifted orthogonal Genocci polynomials and then by using these matrices, fractional delay differential equation is transformed to a system of algebraic equations which can be solved via a suitable numerical method.

Results and discussion

We solve some test examples by using present technique to demonstrate the efficiency, high accuracy and the simplicity of the present method, then compare the proposed method with other methods. The reported results demonstrate that there is a good agreement between approximate solution and exact solution. Also, the numerical results reported in the tables indicate that the accuracy improve by increasing the number of basis functions. In addition, the results show that the method is also effective for long intervals.

Conclusion

The following conclusions were drawn from this research.

- shifted orthogonal Genocci polynomials are simple basis functions, so proposed method is easy to implement and it is a powerful mathematical tool to obtain the numerical solution of various kind of problems with little additional works.
- The main characteristic of this method is that it reduces the underlying problem to a system of algebraic equations which can be easily solved by using direct method or iterative method.
- Convergence analysis shows the convergence of the method, and the method is also effective for long intervals.

Keywords: Fractional delay Volterra integro-differential equations; Genocci polynomials; Galerkin method; Operational matrix; Convergence analysis.

*Corresponding author: mailto:ahmadkhanlu@azaruniv.ac.i babolian@khu.ac.ir