On warped product Finsler metrics with isotropic E-curvature

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Received: 25 February 2020 Accepted: 21 October 2020

Extended Abstract

Paper pages (745-755)

Introduction

The warped product metrics form a rich and an important class of metrics in Riemann-Finsler geometry. For the Riemannian manifolds (M, ds_1^2) and (N, ds_2^2), a warped product is the manifold $M \times N$ endowed with a Riemannian metric of the form

$$ds^2 = ds_1^2 + f^2 ds_2^2,$$

where f is a smooth function depending on the coordinates of M only. Moreover, metrics of such form with arbitrary signature can be easily considered in the realm of pseudo Riemannian geometry. The warped product metric was later extended to the case of Finsler manifolds by the work of Chen-Shen-Zhao and Kozma-Peter-Varga.

In this paper, we mainly study warped product metrics which have been introduced by Chen-Shen-Zhao using the concept of the warped product structure on an n-dimensional manifold $M := I \times \breve{M}$ where I is an interval of R and $\breve{M}$ is an (n-1)-dimensional manifold equipped with a Riemannian metric. In fact, it has been considered in the following form:

$$F(u,v) := \breve{\alpha}(\breve{u},\breve{v})\phi(u^1, \frac{v^1}{\breve{\alpha}(\breve{u},\breve{v})}),$$

where $u = (u^1, \breve{u})$, $v = v^1 \frac{\partial}{\partial u^1} + \breve{v}$ and ϕ is a suitable function defined on a domain of $\mathbb{R}^2$ The class of Finsler metrics called Finsler warped product metrics and includes spherically symmetric Finsler metrics.

Material and methods

In this paper, we first compute the E-curvature of a Finsler warped product metric by the following formula

$$E_{AB} := \frac{1}{2}\frac{\partial^2}{\partial v^A \partial v^B}\left(\frac{\partial G^C}{\partial v^C}\right).$$

Then, we characterize Finsler warped product metrics with isotropic E-curvature with the following formula

$$E = 1/2(n+1)kF^{-1}h,$$

where

$$h_v := h_{AB}du^A \otimes du^B, \quad h_{AB} := FF_{v^A v^B}.$$

Results and discussion

It is known that a Finsler metric F is called a Berwald metric if *its spray* coefficients are quadratic in $y \in T_xM$ for $x \in M$. Taking a trace of Berwald curvature yields mean Berwald curvature E. Computing this curvature and finding its relation with other curvatures in Finsler geometry is very important. Therefore, we first drive a formula for the E-curvature of a Finsler warped product metric. Then, we characterize Finsler warped product metrics with isotropic E-curvature.

Conclusion

The following conclusions were drawn from this research.

- The class of Finsler warped product metric includes spherically symmetric Finsler metrics. So these results are also true for these metrics.
- Finsler warped product metrics with isotropic E-curvature is considered, so one can check the equivalence of this condition with the condition isotropic S-curvature.
- We characterize Finsler warped product metrics with- isotropic E-curvature.

Keywords: Finsler metric, Finsler warped product metric, the E curvature

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