Some Inequalities of Operator Weighted Geometric Mean

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Abstract

In this paper, using the extended Hölder-McCarthy inequality, several inequalities involving the α-weighted geometric mean$(0 \leq \alpha \leq 1)$ of two positive operators are established. In particular, it is proved that if $A, B, X, Y \in \mathcal{B}(\mathcal{H})$ such that A and B are two positive invertible operators, then for all $r \geq 1$,

$$\|X^*(A \#_\alpha B)Y\| \leq \|X^*AX\|^{\frac{1-\alpha}{2}}\|Y^*AY\|^{\frac{1-\alpha}{2}}\|X^*BX\|^{\frac{\alpha}{2}}\|Y^*BY\|^{\frac{\alpha}{2}}$$

and

$$\|X^*(A \#_\alpha B)X\|^r \leq \|\alpha(X^*BX)^r + (1-\alpha)(X^*AX)^r\| - \Omega(X)$$

Where$\#_\alpha$denotes the $\alpha -$ weighted geometric mean and

$$\Omega(X) = \inf_{\|x\|=1}(\sqrt{\langle (X^*BX)^r x, x\rangle} - \sqrt{\langle (X^*AX)^r x, x\rangle})^2 . \min\{\alpha, 1-\alpha\}.$$

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