Numerical Solution of Weakly Singular Ito-Volterra Integral Equations via Operational Matrix Method based on Euler Polynomials

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Extended Abstract

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Introduction

Many problems which appear in different sciences such as physics, engineering, biology, applied mathematics and different branches can be modeled by using deterministic integral equations. Weakly singular integral equation is one of the principle type of integral equations which was introduced by Abel for the first time. These problems are often dependent on a noise source which are neglected due to poor computational tools. So, it is satisfactory to use stochastic models to describe the behaviour of them. Thus, researchers added uncertainty term in the deterministic models and this leads to the stochastic models such as stochastic partial differential equations or stochastic integral equations. Since 1960, by increasing computational power, some random factors are inserted to deterministic integral equations and are created various kinds of stochastic integral equations such as Ito-Volterra integral equations, Ito-Fredholm integral equations, or weakly singular Ito-Volterra integral equations. In more cases, the analytical solution of these equations do not exist or finding their analytic solution is very difficult. Thus, presenting an accurate numerical method is an essential requirement in numerical analysis. Numerical solution of stochastic integral equations because the randomness has its own difficulties. In recent years, some different basis functions have been used to estimate the solution of stochastic integral equations. In this paper, we develop operational matrix method based on Euler polynomials to solve weakly singular Ito-Volterra integral equations. Euler polynomials have received considerable attention in dealing with various problems and equations.

Material and methods

In this scheme, we first calculate operational matrix of integration and stochastic operational matrix of integration based on Euler polynomials and then by using these matrices, weakly singular Ito-Volterra integral equation is transformed to a system of algebraic equations which can be solved via a suitable numerical method.

Results and discussion

We solve some test examples by using present technique to demonstrate the efficiency, high accuracy and the simplicity of the present method, then compare the proposed method with block-pulse method. The reported results demonstrate that there is a good agreement between approximate solution and exact solution. Also, the numerical results reported in the tables indicate that the accuracy improve by increasing the N. Therefore, to get more accurate results, using the larger N is recommended. Note that, obtained results confirm that proposed method enables us to find some more reasonable approximate solutions than block-pulse method.

Conclusion

The following conclusions were drawn from this research.

- Coefficients of the approximate function via Euler polynomials are found very easily and therefore many calculations are reduced.
- Euler polynomials are simple basis functions, so proposed method is easy to implement and it is a powerful mathematical tool to obtain the numerical solution of various kind of problems with little additional works.
- The main characteristic of this method is that it reduces tht considered problem to a system of algebraic equations which can be easily solved by using direct method or iterative method.

Keywords: Stochastic integral equations; Volterra integral equations; Singular integral equations; Operational matrix method; Euler polynomials.

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