Analytical Soliton Solutions Modeling of Nonlinear Schrödinger Equation with the Dual Power Law Nonlinearity

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Extended Abstract

(Paper pages 259-270)

Introduction

In this study, we use a newly proposed method based on the software structure of the maple, called the Khaters method, and will be introducing exponential, hyperbolic, and trigonometric solutions for one of the Schrödinger equations, called the nonlinear Schrödinger equation with the dual power law nonlinearity. Given the widespread use of the Schrödinger equation in physics and engineering, solving this equation is very important with the above method, which includes a large variety of solutions. Schrödinger's nonlinear equation is a partial differential equation that plays a significant role in modern physics. Since quantum mechanics is present in the most modern technologies, such as nuclear energy, computers made of semiconductor materials, lasers, and all quantum phenomena, all the empirical observations of the world around us are consistent with the results of these equations. And this is the Schrödinger equation describing the system of atomic particle motion and instrumentation over time. Hence, because of the importance of the solutions of the Schrödinger equation, which describes many phenomena in physics and engineering, solving this equation is a great necessity. In every phenomenon and process in nature, there are various parameters that are in accordance with the rules governing that phenomenon. The expression of this relation in mathematical language is a functional equation, and the functional equation is derived from a phenomenon in which the tracks of a function change relative to one or several independent variables are studied, called the differential equation. Due to the nature of the Schrödinger's equation, which contains different nonlinear sentences, is of great use in modern sciences, including Quantum Fiery. We can say that the widest range of applications of equations is related to the Schrödinger equation, especially in physics and modern chemistry and quantum electronics. Wherever there are tiny particles, the Schrödinger equation solves the analysis of the most complex issues associated with them.

Material and methods

The framework of this article is that in the second part the Khaters method and the various solutions of this method are discussed briefly. The application of this method is proposed for the following nonlinear Schrödinger equation

$$i\frac{\partial q}{\partial Z}+\frac{1}{2}\frac{\partial^2 q}{\partial T^2}+\left(|q|^{2p}+\upsilon|q|^{4p}\right)q=0,$$

and at the end conclusion is stated.

Results and discussion

We successfully applied the Khaters method to the nonlinear Schrödinger equation and obtained three families of soliton solutions. For example,

Hyperbolic

$$q_{3-1-1}(Z,T)=\Bigg[-\frac{\sqrt{2}}{16}\Big(-5\sqrt{2}\Delta\sqrt{p\upsilon(-5+2p)}+2\sqrt{2}\Delta p\sqrt{p\upsilon(-5+2p)}+$$
$$9\sqrt{\Lambda}\beta p-2\sqrt{\Lambda}\beta p^{2}-10\sqrt{\Lambda}\beta\Big)\Big((p-2)\upsilon\Delta\sqrt{p\upsilon(-5+2p)}\Big)^{-1}+$$
$$\frac{\sqrt{2}}{8}\frac{\sigma\sqrt{p\upsilon\Lambda(-5+2p)}}{\upsilon^{2}p(p-2)\left(8\sigma\alpha p-2\beta^{2}p+3\beta^{2}-12\sigma\alpha\right)}\times$$
$$\left(\frac{-\beta}{2\sigma}-\frac{\sqrt{\beta^{2}-4\alpha\sigma}}{2\sigma}\tanh\left(\frac{\sqrt{\beta^{2}-4\alpha\sigma}}{2}\left(\frac{\beta}{2}\frac{\sqrt{\Lambda}}{\upsilon\Delta}Z-\frac{1}{2}\frac{\sqrt{\Lambda}}{\upsilon\Delta}T+\xi_{0}\right)\right)\right)\Bigg]^{\frac{1}{2p}}e^{i(\alpha Z+\beta T)},$$

Trigonometric

$$q_{3-2-1}(Z,T)=\Bigg[-\frac{\sqrt{2}}{16}\Big(-5\sqrt{2}\Delta\sqrt{p\upsilon(-5+2p)}+2\sqrt{2}\Delta p\sqrt{p\upsilon(-5+2p)}+$$
$$9\sqrt{\Lambda}\beta p-2\sqrt{\Lambda}\beta p^{2}-10\sqrt{\Lambda}\beta\Big)\Big((p-2)\upsilon\Delta\sqrt{p\upsilon(-5+2p)}\Big)^{-1}+$$
$$\frac{\sqrt{2}}{8}\frac{\sigma\sqrt{p\upsilon\Lambda(-5+2p)}}{\upsilon^{2}p(p-2)\left(8\sigma\alpha p-2\beta^{2}p+3\beta^{2}-12\sigma\alpha\right)}\times$$
$$\left(\frac{-\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^{2}-4\alpha\sigma\right)}}{2\sigma}\tan\left(\frac{\sqrt{-\left(\beta^{2}-4\alpha\sigma\right)}}{2}\left(\frac{\beta}{2}\frac{\sqrt{\Lambda}}{\upsilon\Delta}Z-\frac{1}{2}\frac{\sqrt{\Lambda}}{\upsilon\Delta}T+\xi_{0}\right)\right)\right)\Bigg]^{\frac{1}{2p}}e^{i(\alpha Z+\beta T)},$$

Conclusion

The following conclusions were drawn from this paper.

- We obtained new soliton solutions for nonlinear Schrödinger equation with the dual power law nonlinearity.
- The obtained solutions are expressed by hyperbolic functions, trigonometric functions and rational functions. It is shown that the proposed method provide a powerful mathematical tool for solving nonlinear wave equations in mathematical physics and engineering.

Keywords: Nonlinear Schrodinger equation, Maple package, Analytical solutions, Khaters method, Soliton.

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