A Local Strong form Meshless Method for Solving 2D time-Dependent Schrödinger Equations

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Extended Abstract

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Introduction

The time dependent Schrödinger equation with arbitrary potential function is a fundamental equation of quantum mechanics. It is well known that Schrödinger equation play an essential role in the relativistic and non-relativistic quantum mechanics. This equation arises in many other practical domains of physical and technological interest, e.g. optics, seismology and plasma physics. Many problems of solid state physics, atomic and molecular structure and spectra, molecular dynamics and quantum chemistry require the solution of the nonlinear time dependent Schrödinger equation. The nonlinear Schrödinger equation is a Generic model for the slowly varying envelop of a wave-train in conservative, dispersive, mildly nonlinear wave phenomena. It is also obtained as the subsonic limit of the Zakharov model for Langmuir waves in plasma physics. In more cases, the analytical solutions of these equations do not exist or finding their analytic solution is very difficult. Thus, presenting an accurate numerical method is an essential requirement in numerical analysis. In recent years, many papers have been published in the field of meshless methods.

From a point of view they can be classified in two groups: one using a global strategy and another using local strategy. Meshless local Petrov-Galerkin and meshless local integral equation methods can be regarded as the oldest local treatments in the Meshless methods, where both testing and trial approximation are done locally. The Finite collocation (FC) method is another meshless local strategy for which, besides collocation of unknown solution in local subdomains, the PDE governing operator is also enforced in the local collocation systems. Based on the FC approach, this paper proposes a meshless local strategy for the numerical solution of nonlinear time dependent Schrödinger equation.

Material and methods

In the proposed method, the time variable is discretized by a finite difference scheme. Then, a local radial basis function (RBF) method is used for spatial discretization while the PDE governing equation is also imposed in the local systems. The nonlinear term is handled with an iterative approach.

Results and discussion

Some test examples are considered to be solved by using the presented method to demonstrate the efficiency and high accuracy of the method. Two linear and two nonlinear test problems with known exact solutions are considered and then, the simulations to a nonlinear problem with periodic boundary conditions are also presented. The reported results demonstrate that there is a good agreement between approximate solution and exact solution. Also, the numerical results reported in the tables indicate that the accuracy improve by increasing the number of nodal points. Therefore, to get more accurate results, using the larger N are recommended.

Conclusion

The following conclusions were drawn from this research.

- Because of the local treatment of the problem, the main advantage of the method was the well conditioning of the final linear system of equations.
- Due to the use of strong form equation and collocation approach, the method was computationally efficient.
- Radial basis functions are simple basis functions, so proposed method is easy to implement and it is a powerful mathematical tool to obtain the numerical solution of various kind of problems with little additional works in higher dimensional.

Keywords: Local radial basis functions meshless methods, Collocation methods, Finite differences, Schrödinger equation.

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