Infinitely Many Solutions for a Steklov Problem Involving the $p(x)$-Laplacian Operator

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Extended Abstract

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Introduction

Variational problems and elliptic equations with $p(x)$-growth conditions have been widely investigated in latest years, mainly due to their applications in the theory of elastic mechanics. This fruitful generalization of the classical p-Laplace operator is also motivated by questions arising in image restoration as well as in the mathematical modelling of electrorheological fluids. The corresponding functional spaces $L^{p(x)}(\Omega)$ and $W^{\mathrm{m},p(x)}(\Omega)$ enjoy properties similar to those of $L^p(\Omega)$ and $W^{\mathrm{m},p}(\Omega)$, respectively, but there are some crucial differences.

In this paper, by using variational methods and critical point theory for smooth functionals defined on a reflexive Banach space, we establish the existence of infinitely many weak solutions for a Steklov problem involving the $p(x)$-Laplacian depending on two parameters.

Main result

We study the following Steklov problem involving the $p(x)$-Laplacian

$$\begin{cases}\Delta_{p(x)}u = a(x)|u|^{p(x)-2}u & \text{in } \Omega,\\ |\nabla u|^{p(x)-2}\dfrac{\partial u}{\partial \nu} = \lambda f(x,u) + \mu g(x,u) & \text{on } \partial\Omega,\end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, λ is a positive parameter, μ is a nonnegative parameter, $p \in C(\overline{\Omega})$, $\Delta_{p(x)}u := \operatorname{div}(|\nabla u|^{p(x)-2}\nabla u)$ denotes the $p(x)$-Laplacian operator, $f, g\colon \partial\Omega \times \mathbb{R} \to \mathbb{R}$ are Carathéodory functions, $a \in L^\infty(\Omega)$ with $\operatorname{ess\,inf}_\Omega a \geq 0$ and $a \in L^1(\Omega)$ and ν is the outer unit normal to $\partial\Omega$.

Our goal in this paper is to obtain some sufficient conditions to guarantee that problem (1) has infinitely many weak solutions. To this end, we require that the primitive F of f satisfies a suitable oscillatory behavior either at infinity (for obtaining unbounded solutions) or at zero (for finding arbitrarily small solutions), while G, the primitive of g, has an appropriate growth. Our approach is fully variational and the main tool is a general critical point theorem due to Ricceri.

Keywords: $p(x)$-Laplacian operator, Variable exponent Sobolev spaces, Variational methods, Infinitely many solutions.

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