Lie symmetry Analysis and Explicit Exact Dolutions of the Time Fractional Drinfeld-Sokolov-Wilson (DSW) System

Hadi Roohani Ghehsareh*, Ahmad Majlesi, Ali Zaghian;
Department of Mathematics, Malek Ashtar University of Technology, Shahin Shahr, Isfahan, Iran

Received: 22 Oct. 2017 Revised: 19 Feb. 2018

Extended Abstract

(Paper pages 185-200)

Introduction

During recent years, the theory of fractional calculus has gained notable attention from scientists due to its capability for describing various complicated natural phenomena. Complex processes in natural science can be described more exactly by using the theory of fractional calculus. Many anomalous and complicated phenomena in mechanics, chemistry, physics and other sciences have been mathematically modelled as fractional differential and integral equations. The fractional equations are very powerful mathematical tools to describe the inherent characteristics and hereditary properties of materials and processes. In many cases, finding analytical solutions of the fractional differential models is very difficult or sometimes impossible. In the last decade, many researchers have attempted to find and develop efficient numerical and analytical approaches for dealing with fractional differential equations. Recently, the Lie symmetry analysis has been improved and effectively used to deal with various types of nonlinear fractional partial differential equations. It is one of the most powerful techniques to investigate complicated nonlinear fractional models. In this paper, the generalized Lie symmetry approach proposed by Singla and Gupta is used for symmetry analysis of the time fractional Drinfeld-Sokolov-Wilson (DSW) system. The time fractional Drinfeld-Sokolov-Wilson (DSW) system is a challenging and anomalous physical model which describes nonlinear surface gravity waves propagating over horizontal seabed. The invariant subspace method is an efficient and powerful analytical approach for finding explicit particular solutions of nonlinear partial differential equations. Recently, the method has been developed by Gazizov et al. for fractional differential equations. Very recently, the method has been extended by Sahadevan and Prakash for solving a coupled system of time-fractional partial differential equations. In this study, the recent procedure is performed to find a set of explicit particular solutions of the time fractional Drinfeld-Sokolov-Wilson (DSW) system.

Material and methods

In this study, a challenging and anomalous physical model which describes nonlinear surface gravity waves propagating over horizontal seabed, the time fractional Drinfeld-Sokolov-Wilson (DSW) system, is investigated. A fractional Lie symmetry analysis is used to investigate invariance properties and obtain Lie point symmetries, similarity variables, similarity transformations of the problem. The governing nonlinear system of time fractional partial differential equations is similarity reduced to a system of nonlinear ordinary differential

equations with Erdelyi-Kober fractional derivatives. Finally, by implementing the invariant subspace method, a set of explicit particular solutions for the problem has been well constructed.

Results and discussion

The Lie point symmetries of the time fractional Drinfeld-Sokolov-Wilson (DSW) system are derived by implementing the generalized Lie symmetry analysis. Using the obtained Lie point symmetries, the corresponding similarity variables, similarity transformations and invariant solutions of the nonlinear fractional model are extracted. The nontrivial invariant solutions are utilized to similarity transform the considered nonlinear time fractional partial differential system into a fractional system of ordinary differential equations with the Erdelyi-Kober fractional derivatives. Moreover, a set of explicit particular solutions of the time fractional Drinfeld-Sokolov-Wilson (DSW) system is obtained by formulating and implementing an extended invariant subspace method. The results demonstrate the high ability and efficiency of the proposed approaches to deal with complicated physical models.

Conclusion

This paper aims to study the time fractional Drinfeld-Sokolov-Wilson (DSW) system via Lie symmetry analysis and invariant subspace method. The time fractional DSW system is an important physical model for describing nonlinear surface gravity waves propagating over horizontal seabed. The generalized Lie symmetry analysis is successfully performed to derive the Lie point symmetries of the problem. Using the derived nontrivial symmetry generators, the governing nonlinear time-fractional system of partial differential equations is similarity reduced into a fractional system of ordinary differential equations involving the extended Erdelyi-Kober fractional differential operators. Also, a class of analytical particular solutions of the time fractional DSW system has been explicitly computed by using the concept of invariant subspaces. The derived analytical particular solutions might be very useful to realize the model.

Keywords: Lie symmetry analysis, Time fractional Drinfeld-Sokolov-Wilson (DSW) system, Erdelyi-Kober operators, Invariant subspace method.

*Corresponding author: roohani@mut-es.ac.ir