Existence Results of best Proximity Pairs for a Certain Class of Noncyclic Mappings in Nonreflexive Banach Spaces Polynomials

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Received: 4 April 2017 Revised: 3 April 2018

Extended Abstract

(Paper pages 229-240)

Introduction

Let A be a nonempty subset of a normed linear space X. A self-mapping $T: A \to A$ is said to be nonexpansive provided that $||Tx - Ty|| \leq ||x - y||$ for all $x, y \in A$. In 1965, Browder showed that every nonexpansive self-mapping defined on a nonempty, bounded, closed and convex subset of a uniformly convex Banach space X, has a fixed point. In the same year, Kirk generalized this existence result by using a geometric notion of normal structure. We recall that a nonempty and convex subset A of a Banach space X is said to have normal structure if for any nonempty, bounded, closed and convex subset K of A with $diam(K) > 0$, there exists a point $p \in K$ for which $\sup\{||p - x||: x \in K\} < diam(K)$. The well-known Kirk's fixed point theorem states that if A is a nonempty, weakly compact and convex subset of a Banach space X which has the normal structure and $T: A \to A$ is a nonexpansive mapping, then T has at least one fixed point. In view of the fact that every nonempty, bounded, closed and convex subset of a uniformly convex Banach space X has the normal structure, the Browder' fixed point result is an especial case of Kirk's theorem.

Material and methods

Let (A, B) be a nonempty pair of subsets of a normed linear space X. $T: A \cup B \to A \cup B$ is said to be a noncyclic mapping if $T(A) \subseteq A,\ T(B) \subseteq B$. Also the noncyclic mapping T is called relatively nonexpansive whenever $||Tx - Ty|| \leq ||x - y||$ for any $(x, y) \in A \times B$. Clearly, if $A = B$, then we get the class of nonexpppansive self-mappings. Moreover, we note the noncyclic relatively nonexpansive mapping T may not be continuous, necessarily. For the noncyclic mapping T, a point $(p, q) \in A \times B$ is called a best proximity pair provided that

$$p = Tp, \quad q = Tq, \quad ||p - q|| = \text{dist}(A, B).$$

In the other words, the point $(p, q) \in A \times B$ is a best proximity pair for T if p and q are two fixed points of T which estimates the distance between the sets A and B.

The first existence result about such points which is an interesting extension of Browder's fixed point theorem states that if (A, B) is a nonempty, bounded, closed and convex pair in a uniformly convex Banach space X and if $T: A \cup B \to A \cup B$ is a noncyclic relatively nonexpansive mapping, then T has a best proximity pair. Furthermore, a real generalization of Kirk's fixed point result for noncyclic relatively nonexpansive mappings was proved by using a geometric concept of proximal normal structure, defined on a nonempty and convex pair in a considered Banach space.

Results and discussion

Let (A, B) be a nonempty and convex pair of subsets of a normed linear space X and $T: A \cup B \to A \cup B$ be a noncyclic mapping. The main purpose of this article is to study of the existence of best proximity pairs for another class of noncyclic mappings, called noncyclic strongly relatively C-nonexpansive. To this end, we use a new geometric notion entitled T-uniformly semi-normal structure defined on (A, B) in a Banach space which is not reflexive, necessarily. To illustrate this geometric property, we show that every nonempty, bounded, closed and convex pair in uniformly convex Banach spaces has T-uniformly semi-normal structure under some sufficient conditions.

Conclusion

The following conclusions were drawn from this research.

We introduce a geometric notion of T-uniformly semi-normal structure and prove that: Let (A, B) be a nonempty, bounded, closed and convex pair in a strictly convex Banach space X such that A_0 is nonempty and $\text{dist}(A, B) > 0$. Let $T: A \cup B \to A \cup B$ be a noncyclic strongly relatively C-nonexpansive mapping. If (A, B) has the T-uniformly semi-normal structure, then T has a best proximity pair.

In the setting of uniformly convex in every direction Banach space X, we also prove that: Let (A, B) be a nonempty, weakly compact and convex pair in X and $T: A \cup B \to A \cup B$ be a noncyclic mapping such that $\|Tx - Ty\| = \|x - y\|$, for all $(x, y) \in A \times B$ with $\|x - y\| = \text{dist}(A, B)$. If

$$\inf\{\|x - T\mathcal{P}x\| : x \in A_0 \cup B_0\} > dist\,(A, B),$$

where $\mathcal{P}$ is a projection mapping defined on $A_0 \cup B_0$, then (A, B) has T-semi-normal structure.
We present some examples showing the useability of our main conclusions.

Keywords: Strongly relatively C-nonexpansive mapping, Best proximity pair, Uniformly convex space, T-Uniformly semi-normal structure.

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