Preconditioned Generalized Minimal Residual Method for Solving Fractional Advection-Diffusion Equation

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Extended Abstract

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Introduction

Fractional differential equations (FDEs) have attracted much attention and have been widely used in the fields of finance, physics, image processing, and biology, etc. It is not always possible to find an analytical solution for such equations. The approximate solution or numerical scheme may be a good approach, particularly, the schemes in numerical linear algebra for solving a system, , emerging by discretizing the partial derivatives, with large and sparse dimensions. In the procedure of solving a specified FDE, if the dimension of the corresponding system of linear equations is small, one can use the direct methods or the classical iterative methods for the analysis of these systems. However, if the dimension is large, then the proposed methods are not effective. In this case, we use variants of the Krylov subspace methods that are more robust with respect to the computer memory and time. The GMRES (Generalized Minimal Residual) is a well-known method based on Krylov subspace that is used to solve a system of sparse linear equations with an non-symmetric matrix. A main drawback of iterative methods is the slowness of convergence rate which depends on the condition number of the corresponding coefficient matrix. If the condition number of the coefficient matrix is small, then the rate of convergence will be faster. So, we try to convert the original system to another equivalent system, in which the condition number of its coefficient matrix becomes small. A preconditioner matrix is a matrix that performs this transformation.

In this paper, we propose the iterative GMRES method, preconditioned GMRES method and examine capability of these methods by solving the space fractional advection-diffusion equation.

Material and methods

We first introduce a space fractional advection-diffusion equation in the sense of the shifted Grünwald-Letnikov fractional derivative. To improve the introduced numerical scheme, we discretize the partial derivatives of equation using the fractional Crank-Nicholson finite difference method. Then we use a preconditioner matrix and present preconditioned GMRES method for solving the derived linear system of algebraic equations.

Results and discussion

In this paper, we use the GMRES and preconditioned GMRES to solve a linear system of equations emerging by discretizing partial derivatives appearing in a Advection-Diffusion

equation and then asset the accuracy of these methods. Numerical results indicate that we derive a smaller condition number of the equivalent coefficient matrix for different values of M and N, as dimensions of the corresponding linear equations. Hence the convergence rate increases and consequently the number of iterations and the calculation time decreases.

Conclusion

The following conclusions were drawn from this research.

- The GMRES method is a Kyrlov subspace methods to solve large-dimensions non-symmetric system of linear equations, which will be more effective when is applied with preconditioning techniques.
- One of the common ways to increase the rate of convergence of iterative methods based on the Kyrlov subspace is the applying the preconditioned techniques.
- An appropriate preconditioner matrix increases the rate of convergence of the iterative method.

Keywords: Fractional advection-diffusion equation, Fractional Crank-Nicholson method, Shifted Grünwald-Letnikov fractional derivative, Preconditioned GMRES method.

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