Approximate Duals Introduced by Multipliers and Their Role in the Reconstruction of Signals

Morteza Mirzaee Azandaryani*, Mehdi Rahimi
Department of Mathematics, University of Qom, Qom, Iran

Received: 20 July2017 Accepted: 17 Sep 2018

Extended Abstract

Paper pages (237-252)

Introduction

Frames for Hilbert spaces were first introduced by Duffin and Schaeffer in 1952 to study some problems in nonharmonic Fourier series, reintroduced in 1986 by Daubechies, Grossmann and Meyer. Various generalizations of frames have been introduced and many applications of them in different branches have been presented.

Bessel multipliers in Hilbert spaces were introduced by Peter Balazs. As we know in frame theory, the composition of the synthesis and analysis operators of a frame is called the frame operator. A multiplier for two Bessel sequences is an operator that combines the analysis operator, a multiplication pattern with a fixed sequence, called the symbol, and the synthesis operator. Bessel multipliers have useful applications, for example they are used for solving approximation problems and they have applications as time-variant filters in acoustical signal processing. We mention that many generalizations of Bessel multipliers have been introduced, also multipliers have been studied for non-Bessel sequences.

Approximate duals in frame theory have important applications, especially are used for the reconstruction of signals when it is difficult to find alternate duals. Approximate duals are useful for wavelets, Gabor systems and in sensor modeling. Approximate duality of frames in Hilbert spaces was recently investigated by Christensen and Laugesen and some interesting applications of approximate duals were obtained. For example, it was shown that how approximate duals can be obtained via perturbation theory and some applications of approximate duals to Gabor frames especially Gabor frames generated by the Gaussian were presented. Afterwards, many authors studied approximate duals of Bessel sequences and many properties and generalizations of them were presented. In this note, we consider approximate duals for arbitrary sequences.

Results and discussion

In this paper, we introduce some new kinds of duals and approximate duals in Hilbert spaces using multipliers, invertible operators and symbols. Many papers about approximate duals and their applications have been written so far which in these papers approximate duals have been considered for Bessel sequences. Here, we introduce approximate duals for arbitrary sequences in a Hilbert space, compare them with Bessel approximate duals and we show that they can be useful for the reconstruction of signals though they do not have all of the properties of Bessel approximate duals. Moreover, we obtain some new results for Bessel approximate duals.

Conclusion

The following conclusions were drawn from this research.

- New kinds of duals and approximate duals for arbitrary sequences are introduced using multipliers, invertible operators and symbols.
- Duals and approximate duals of non-Bessel sequences are compared with the Bessel ones and some differences between them are shown by presenting various examples.
- Some properties and applications of duals and approximate duals of non-Bessel sequences are stated.
- Some new results about duals and approximate duals of Bessel sequences are obtained especially some important concepts such as closeness of Bessel sequences, nearly Parseval frames and multipliers with constant symbols are related to approximate duals of frames.

Keywords: Hilbert space, Bessel sequence, Frame, Approximate dual, Multiplier, Reconstruction of signals.

*Corresponding author: morteza_ma62@yahoo.com