A Short Proof of the Maximum Conjecture in CR Dimension one

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Extended Abstract

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Introduction

In this paper and by employing some certain techniques and results arisen in the theory of Tanaka, we provide a short proof for the maximum conjecture on the rigidity of Beloshapka's models of the specific CR dimension one and of length ≥ 3. As a consequence, we realize that the Lie group of biholomorphisms (CR automorphisms) associated with each of these models only consists of linear maps.

Material and methods

Two major approaches employed so far to study Beloshapka's maximum conjecture have been 1) the approach of envelope of holomorphy, applied by Ilya Kossovskii to confirm the mentioned conjecture in the specific lengths three and four and 2) Cartan's approach of solving (biholomorphic) equivalence problems, employed by the present author in CR dimension one. Both of these approaches are *geometric*. In this paper, we introduce and employ the *algebraic* approach of applying some techniques in Tanaka's theory of transitive prolongations to study the already mentioned conjecture in the specific CR dimension one. The proofs are based upon some results concerning Tanaka prolongation of rank two fundamental algebras of lengths ≥ 3, achieved by Medori and Nacinovich (1997). Here, we prove first an *algebraic parallel version* of Beloshapka's conjecture and employ the results to solve this geometric open problem in CR dimension one.

Results and discussion

As is the main goal of this paper, we confirm the maximum conjecture on the rigidity of Beloshapka's CR models in CR dimension one. As a consequence, we realize that each biholomorphic deformation of these models is linear. It may be worth to notice that the maximum conjecture in CR dimension one has been confirmed before by the present author, using the Cartan approach of solving equivalence problems. But, here we provide a much shorter proof for this result.

Conclusion

The following conclusions were drawn from this research.

- We confirm Beloshapka's maximum conjecture in CR dimension one.
- It is introduced a new approach to consider this conjecture.

- Comparing the performance of the Tanaka approach, employed in this paper, with two approaches of "envelope of holomorphy" and "Cartan's theory", this algebraic approach seems the best weapon, introduced so far, to attack the maximum conjecture.

Keywords: Totally nondegenerate CR models, Maximum conjecture, Tanaka prolongation.

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