Gorenstein Injective Dimension and Cohen-Macaulayness

Reza Sazeedeh[*], Fatemeh Savoji

Department of Mathematics, Science Faculty, Urmia University, Urmia, Iran

Extended Abstract

Throughout this paper, (R, m) is a commutative Noetherian local ring with the maximal ideal **m**. The following conjecture proposed by Bass [1], has been proved by Peskin and Szpiro [2] for almost all rings:

(B) *If R admits a finitely generated R-module of finite injective dimension, then R is Cohen-Macaulay.*

The problems treated in this paper are closely related to the following generalization of Bass conjecture which is still wide open:

(GB) *If R admits a finitely generated R-module of finite Gorenstein-injective dimension, then R is Cohen-Macaulay.*

Our idea goes back to the first steps of the solution of Bass conjecture given by Levin and Vasconcelos in 1968 [3] when R admits a finitely generated R-module of injective dimension ≤ 1.

Levin and Vasconcelos indicate that if $x \in \mathrm{m} \backslash \mathrm{m}^2$ is a non-zerodivisor, then for every finitely generated R/xR-module M, there is $\mathrm{id}_R M = \mathrm{id}_{R/xR} M + 1$. Using this fact, they construct a finitely generated R-module of finite injective dimension in the case where R is Cohen-Macaulay (the converse of Conjecture **B**).

In this paper we study the Gorenstein injective dimension of local cohomology. We also show that if R is Cohen-Macaulay with minimal multiplicity, then every finitely generated module of finite Gorenstein injective dimension has finite injective dimension.

We prove that a Cohen-Macaulay local ring has a finitely generated module of finite Gorenstein injective dimension.

Keywords: Gorenstein injective, Cohen-Macaulay ring and Local cohomology modules

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[*]Corresponding author: rsazeedeh@ipm.ir