Bishop-Phelps Theorem for Normed Cones

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Received: 21 Feb. 2017 Accepted: 23 April 2018

Extended Abstract

Paper pages (165-174)

Introduction

In the last few years there is a growing interest in the theory of quasi-metric spaces and other related structures such as quasi-normed cones and asymmetric normed linear spaces, because such a theory provides an important tool in the study of several problems in theoretical computer science, approximation theory, applied physics, convex analysis and optimization. Many works on general topology and functional analysis have recently been obtained in order to extend the well-known results of the classical theory of normed linear spaces to the framework of asymmetric normed linear spaces and quasi-normed cones.

An abstract cone is analogous to a real vector space, except that we take $\mathbb{R}_+$ as the set of scalars. In 2004, O. Valero introduced the normed cones and proved some closed graph and open mapping results for normed cones. Also Valero defined and studied some properties of quotient normed cones. P. Selinger studied the norm properties of a cone with its order properties and proved Hahn-Banach theorems in these cones under the appropriate conditions. Valero and his colleagues discussed the metrizability of the unit ball of the dual of a normed cone and the isometries of normed cones. Other properties are investigated in a series of papers by Romaguera, Sanchez Perez and Valero.

The Bishop-Phelps theorem is a fundamental theorem in functional analysis which has many applications in the geometry of Banach spaces and optimization theory. The classical Bishop-Phelps theorem states that "the set of support functionals for a closed bounded convex subset B of a real Banach space X, is norm dense in X^* and the set of support points of B is dense in the boundary of B". Indeed, E. Bishop and R. R. Phelps answer a question posed by Victor Klee in 1958. We give an analogue to the normed cones, in fact we show that in a continuous normed cone the set of support points of a closed convex set is a dense subset of the boundary under the appropriate hypothesis.

Conclusion

In this paper the notion of support points of convex sets in normed cones is introduced and it is shown that in a continuous normed cone, under the appropriate conditions, the set of support points of a bounded Scott-closed convex set is nonempty. We also present a Bishop-Phelps type Theorem for normed cones.

Keywords: Support point; Normed cone; Bishop-Phelps theorem

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