The Automorphism Group of Non-Abelian Group of Order p^4

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Abstract

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Introduction

Determining the order and the structure of the automorphism group of a finite p-group is an important problem in group theory. There have been a number of studies of the automorphism group of p-groups. Most of them deal with the order of $Aut(G)$, the automorphism group of G,. Moreover various attempts have been made to find a structure for the automorphism group of a finite p-group. Following [2], [29], two groups are isoclinic if their commutator subgroups and central quotients are isomorphic and their commutator operations are essentially the same. We use the classification of p-groups by James [5], which based on isoclinism. By using [5] we have ten non-isoclinic families of non-abelian groups of order p^4.

Let G be a finite non-abelian group of order p^4 , $Aut_p(G)$ be the Sylow p-subgroup of $Aut(G)$ and $\phi = \phi(G)$ be the Frattini subgroup of G. It is well-known [7 , Satz III. 3.17] that the $Aut^{\Phi}(G)$, the group of all automorphisms of G centralizing $\frac{G}{\phi(G)}$, is normal p-subgroup of $Aut(G)$. In this paper we give a structure theorem for $Aut_p(G)$. Also we prove that if G is p-group of maximal class then $Aut_p(G) = Aut^{\Phi}(G)$ or $Aut_p(G)$ is a split extension of $Aut^{\Phi}(G)$ by a cyclic group of order p.

Keywords: Automorphism group, Central Automorphisms, finite p-group.

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