High-Order Jacobi Collocation Method for the Single Term Nonlinear Fractional Differential Equations

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Extended Abstract

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Introduction

The modeling of many real-life physical systems leads to a set of fractional differential equations. Also fractional differential equations appear in various physical processes such as viscoelasticity and viscoplasticity, modeling of polymers and proteins, transmission of ultrasound waves, signal processing, control theory, etc. Most of fractional differential equations especially their nonlinear types do not have exact analytic solution, so numerical methods must be used. Therefore many authors have worked on the numerical solutions of this kind of equations. In recent years, many numerical methods have emerged, such as, the Adomian decomposition method, the Homotopy method, the multistep method, the extrapolation method, the spline collocation method, the product integration method and the predictor-corrector method. But most of the aforementioned methods consider the linear type of equations without a reliable theoretical justification. Then providing an efficient numerical scheme to approximate the solutions of nonlinear fractional differential equations is worthwhile and new in the literature. The main object of this paper is to develop and analyze a high order numerical method based on the collocation method when applies the orthogonal Jacobi polynomials as bases functions for the single term nonlinear fractional differential equations.

Material and methods

Due to the well-known existence and uniqueness theorems the solutions of the fractional differential equations typically suffer from singularity at the origin. Consequently direct application of the Jacobi collocation method may lead to very weak numerical results. To fix this difficulty, we introduce a smoothing transformation that removes the singularity of the exact solution and enables us to approximate the solution with a satisfactory accurate result. Convergence analysis of the proposed scheme is also presented which demonstrates that the regularization process improves the smoothness of the input data and thereby increases the order of convergence.

Results and discussion

We illustrate some test problems to show the effectiveness of the proposed scheme and to confirm the obtained theoretical predictions. In overall, the reported results justify that the proposed regularization strategy works well and the obtained approximate solutions have a good accuracy. To show the applicability of our approach we solve a practical example which is developed for a micro-electrical system (MEMS) instrument that has been designed primary to measure the viscosity of fluids that are encounter during oil well exploration using the proposed scheme. Moreover, we make a comparison between our scheme and the operational Tau method to show the efficiency of our technique. The reported results approve the superiority of the proposed approach. Finally, we consider a problem that we do not have access to its exact solution. In this case, we use the “Variational Iteration Method (VIM)” as a qualitatively correct picture of the exact solution (the source solution) to evaluate the precision of the proposed technique. The obtained results approve that our approach produces the approximate solution which is in a good agreement with source ones.

Conclusion

The following conclusions were drawn from this research.

- A reliable numerical method based on the Jacobi collocation method to approximate the solutions of a class of nonlinear fractional differential equations was developed.
- To achieve an efficient approximation a regularization strategy was proposed that improves the smoothness of the input data and enables us to obtain an approximate solution with a satisfactory accuracy.
- Convergence analysis of the proposed method was investigated which confirmed the high order of convergence of the proposed method.

Keywords Single term nonlinear fractional differential equations; Collocation method; Jacobi polynomials; Convergence analysis.

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