

Group Rings Satisfying Generalized Engel Conditions

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Received: May 2017

Accepted: Nov. 2018

Extended Abstract

Paper pages (57-64)

Introduction

Let R be a ring. For each pair of elements x_1 and x_2 in R denote $[x_1, x_2] = x_1x_2 - x_2x_1$. Define inductively $[x_1, \dots, x_{n+1}] = [[x_1, \dots, x_n], x_{n+1}]$ for each natural number n . We say that R is a *Lie nilpotent* ring if there exists a natural number n such that for each elements $a_1, a_2, \dots, a_n \in R$, we have $[a_1, \dots, a_n] = 0$. Also, the ring R is called *Lie Engel* if for each $a, b \in R$, there exists a natural number $n = n(a, b)$

$$[a, {}_n b] := [a, \underbrace{b, \dots, b}_{\text{بار } n}] = 0.$$

If n can be chosen independently such that for all $a, b \in R$ we have $[a, {}_n b] = 0$, then R is said a *bounded Lie Engel* ring. Clearly, every Lie nilpotent ring is a bounded Engel ring but the converse in general is not true.

We say that the ring R is *generalized Lie Engel* if for each $a, b \in R$, there exist natural numbers $n = n(a, b)$ and $m = m(a, b)$ such that $[a^m, {}_n b] = 0$.

Let R be a commutative unitary ring of characteristic $r \geq 0$ and let G be a group. It is known that the group ring RG is a bounded Lie Engel ring if and only if either G is a nilpotent group and G contains a subgroup A such that both of G/A and A' are finite p -groups provided that r is a power of a prime p , or G is abelian provided r is not a power of a prime or $r = 0$. In this paper we show that if G is a locally finite group, then RG is a generalized Lie Engel ring if and only if either G is locally nilpotent and G' is a p -group if r is a power of a prime p , or G is abelian if r is not a power of a prime or $r = 0$.

Theorem. *Let R be a commutative unitary ring of characteristic $r \geq 0$ and G be a locally finite group. Necessary and sufficient conditions for RG to be a generalized Lie Engel ring are:*

Also, we show that if F is a field and G is a finite group, then FG is a generalized Lie Engel ring if and only if FG is a Lie nilpotent ring.

In this paper, also, we define some generalized Engel conditions on groups, and then we present a result about unit group of group algebras which satisfies this kinds of generalized Engel conditions.

Keywords: Group rings; Engel groups; Generalized Lie Engel rings.

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