Contact CR Submanifolds of maximal Contact CR dimension of Sasakian Space Form

Mohammad Ilmakchi^{*}, Esmaiel Abedi

Department of Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran

Received: July 2017 Accepted: September 2018 Extended Abstract

Paper pages (1-12)

Introduction

Let M be a (connected) n+1-dimensional submanifold of a Sasakian space form $(\overline{M}(c), \phi, \xi, \eta, g)$. Then it is known that if the maximal ϕ -invariant subspace of each tangent space is n-1-dimensional, M admits a naturally induced metric structure. For the hypersurface case, the maximal ϕ -invariant subspace is necessarily of dimension n-1.

Kim, et al. studied the maximal dimensional contact CR-submanifold in unit sphere which satisfy the condition

$$h(FX, Y) + h(X, FY) = 0$$

and determined such submanifolds under the additional condition, where F denotes a skew-symmetric endomorphism induced from ϕ acting on the tangent bundle TM and h the second fundamental form on M.

Also Okunura, et al. studied the maximal dimensional contact CR-submanifold in complex space form with same condition.

Afterward they studied the maximal dimensional contact CR-submanifold in unit sphere and complex space form which satisfy the condition

$$h(FX, Y) + h(X, FY) = g(FX, Y)\zeta$$

for a normal vector field ζ to M .

Recently Kim and etc. studied the maximal dimensional contact CR-submanifold in Sasakian space form which satisfy the condition

$$h(FX, Y) + h(X, FY) = 0.$$

In this paper we study n+1-dimensional contact CR-submanifold of n-1 contact CR-dimension in a Sasakian space form and determine such submanifolds in a complete simply connected Sasakian space form under assumption that

$$h(FX, Y) + h(X, FY) = g(FX, Y)\zeta$$

Preliminary

In this paper, we investigate contact CR submanifolds of contact CR dimension in Sasakian space form and introduce the general structure of these submanifolds and then studying structures of these submanifolds with the condition

 $h(FX, Y) + h(X, FY) = g(FX, Y)\zeta$,

for the normal vector field ζ , which is nonzero, and we classify these submanifolds.

Results and discussion

We show, for Contact CR Submanifolds of maximal Contact CR dimension M of Sasakian Space Form $\overline{M}(c)$, the following results:

Let M be a Contact CR Submanifolds of maximal Contact CR dimension of Sasakian Space Form $\overline{M}(c)$ with above condition then the eigenvaluees of the shape operator A are constant.

Let M be a Contact CR Submanifolds of maximal Contact CR dimension of Sasakian Space Form $\overline{M}(c)$ with above condition then the shape operator A has 2 or 3 or 4 eigenvalues. If the shape operator A has 2 eigenvalues then the multiplicities of them are 1 and n. If the shape operator A has 3 eigenvalues then the multiplicities of them are 1, 1 and n-1. If the shape operator A has 4 eigenvalues then the multiplicities of them are 1, 1, $\frac{n-1}{2}$ and $\frac{n-1}{2}$.

Conclusion

The following conclusions were drawn from this research.

Let M be a Contact CR Submanifolds of maximal Contact CR dimension of Sasakian Space Form $\overline{M}(c)$ with above condition then:

- *M* is locally a product $S^1 \times M'$, where *M'* belongs to some sphere of odd-dimension.
- *M* is locally a product $M' \times C$, where *C* geodesic curve and *M'* totally geodesic hypersurface are in *M*.
- *M* is locally a product $M_1 \times M_2$, where M_1 and M_2 are submanifolds in *M*.

Keywords: Sasakian space form; Contact CR Submanifolds of maximal Contact CR dimension; submanifold.

*Corresponding author: ilmakchi@azaruniv.ac.ir