Solving Optimal Control Problems with Integral Equations or Integral-Differential Equations using Cubic Scale Functions and B-Spline Wavelets

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Extended Abstract

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Introduction

Optimal control problems (OCPs) appear in a wide class of applications. In the classical control problems, the state-space equations are expressed as differential equations. Many physical systems, technology, biology, viscoelastic, electrochemical, economic, and generally the systems that have a memory effect cannot properly be described as ordinary differential equations. Hence, the equation of these systems expresses as integral equations, integro-differential equations, fractional differential equations and fractional integro-differential equations. Almost every system of controlled ordinary differential equations or controlled integro-differential equations can be modeled by a class of systems of controlled Volterra integral equations. There are many methods for solving optimal control problems with the state space of the system in the form of ordinary, fractional, and integral equations; can be mentioned the Euler-Lagrange method, the method of using Pontryagin's maximum principle, the numerical methods based on finite difference, finite element methods, conjugate gradient method, spectral methods, the methods of continuous orthogonal functions, the operational matrices of integrals and embedding method. The method which we used in this paper is based on using the operational matrix of cubic B-spline scaling functions and wavelets with collocation method to reduce the optimal control problem governed by the nonlinear integral equation and integro-differential equation system with quadratic performance index to a nonlinear programming. The semi-orthogonal B-spline scaling functions and wavelets and their dual functions used in this paper have compact support, vanishing moments. These properties make many of the operational matrix elements be very small compared with the largest ones. These scaling functions and wavelets can be represented in a closed form so working with them is easy. The convergence of control and state functions and the performance index of the optimal approximation of the proposed method and also the upper bound of the error are given.

Material and methods

In this paper, a numerical method based on cubic B-spline scaling functions and wavelets for solving optimal control problems with the dynamical system of the integral equation or the differential-integral equation is discussed. The Operational matrices of derivative and integration of the product of two cubic B-spline wavelet vectors, collocation method and Gauss-Legendre integration rule for the discretization of the continuous optimal control problem and its transformation into a problem of non-linear programming is used.

Results and discussion

We solve two examples of optimal control with the dynamical system of integral equation and two examples with the dynamical system of the integro-differential equation by using present method to demonstrate validity, applicability and the simplicity of the new technique, then compare the present method with hybrid pseudo-spectral and Legendre wavelets method. These results illustrate that the accuracy of our numerical solutions are a few better than the numerical solutions obtained in the other method and there is a good agreement between the approximate solution and exact solution. Also, the numerical results reported in the tables and convergence analysis demonstrate that the accuracy improve by increasing the j_u. Therefore, to get more accurate results, using the larger j_u is recommended.

Conclusion

The following conclusions were drawn from this research.

- The operation matrix can be simply obtained for any basis of the approximation space and it is always available, therefore it can be applied to obtain the numerical solution of various kind of optimal control problems.
- Numerical results and convergence analysis indicates that the approximation solution fairly matches with the exact solution and the upper bound of error exponentially decreases by growing of approximation space.
- Due to the characteristics of the B-spline wavelet and dual of them, a nonlinear objective function can be obtained without calculating the integral.
- The semi-orthogonal B-spline scaling functions and wavelets used in the present paper have the properties of compact support, vanishing moments, smoothness function and the representation by a closed-form expression. With these assumptions, time is reduced, computer memory is less occupied and the operation matrix is always available.

Keywords: Operational matrix, Optimal control problems, Integral equations, Cubic scale functions and B-spline wavelets, Collocation methods, Numerical integration.

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