Group Rings Satisfying Generalized Engel Conditions

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Extended Abstract
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Introduction

Let \( R \) be a ring. For each pair of elements \( x_1 \) and \( x_2 \) in \( R \) denote \([x_1, x_2] = x_1x_2 - x_2x_1\). Define inductively \([x_1, \ldots, x_n] = [[x_1, \ldots, x_n], x_{n+1}]\) for each natural number \( n \). We say that \( R \) is a Lie nilpotent ring if there exists a natural number \( n \) such that for each elements \( a_1, a_2, \ldots, a_n \in R \), we have \([a_1, \ldots, a_n] = 0\). Also, the ring \( R \) is called Lie Engel if for each \( a, b \in R \), there exists a natural number \( n = n(a, b) \)

\([a, n b] := [a, b, \ldots, b] = 0\).

If \( n \) can be chosen independently such that for all \( a, b \in R \) we have \([a, n b] = 0\), then \( R \) is said a bounded Lie Engel ring. Clearly, every Lie nilpotent ring is a bounded Engel ring but the converse in general is not true.

We say that the ring \( R \) is generalized Lie Engel if for each \( a, b \in R \), there exist natural numbers \( n = n(a, b) \) and \( m = m(a, b) \) such that \([a^m, n b] = 0\).

Let \( R \) be a commutative unitary ring of characteristic \( r \geq 0 \) and let \( G \) be a group. It is known that the group ring \( RG \) is a bounded Lie Engel ring if and only if \( G \) is a nilpotent group and \( G \) contains a subgroup \( A \) such that both of \( G/A \) and \( A' \) are finite \( p \)-groups provided that \( r \) is a power of a prime \( p \), or \( G \) is abelian provided \( r \) is not a power of a prime or \( r = 0 \). In this paper we show that if \( G \) is a locally finite group, then \( RG \) is a generalized Lie Engel ring if and only if either \( G \) is locally nilpotent and \( G' \) is a \( p \)-group if \( r \) is a power of a prime \( p \), or \( G \) is abelian if \( r \) is not a power of a prime or \( r = 0 \).

Theorem. Let \( R \) be a commutative unitary ring of characteristic \( r \geq 0 \) and \( G \) be a locally finite group. Necessary and sufficient conditions for \( RG \) to be a generalized Lie Engel ring are:

Also, we show that if \( F \) is a field and \( G \) is a finite group, then \( FG \) is a generalized Lie Engel ring if and only if \( FG \) is a Lie nilpotent ring.

In this paper, also, we define some generalized Engel conditions on groups, and then we present a result about unit group of group algebras which satisfies this kinds of generalized Engel conditions.

Keywords: Group rings; Engel groups; Generalized Lie Engel rings.

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