A Spectral Method Based on Hahn Polynomials for Numerical Solution of Fractional Integro-Differential Equations with Weakly Singular Kernel

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Received: Nov. 2017 Accepted: June 2018

Extended Abstract

Paper pages (65-78)

Introduction

Despite wide applications of constant order fractional derivatives, some systems require the use of derivatives whose order changes with respect to other parameters. Samko and Ross produced an extension of the classical fractional calculus with a continuously varying order for differential and integral operators. Variable-order fractional (V-OF) calculus has applications in optimal control, processing of geographical data, diffusion processes, description of anomalous diffusion, heat-transfer problems, etc. Due to the V-OF operators which are non-local with singular kernels, finding the exact solutions of V-OF problems is difficult. Therefore, efficient numerical techniques are necessary to be developed. The numerical solution of V-OF differential equation has been considered in some papers.

Recently, discrete orthogonal polynomials have been considered as basis functions instead of continuous orthogonal polynomials. Discrete orthogonal polynomials are orthogonal with respect to a weighted discrete inner product. These polynomials have important applications in chemical engineering, theory of random matrices, queuing theory and image coding. In this paper, we focus on a special class of discrete polynomials, called Hahn polynomials.

In this work, first, a new operational matrix is obtained for V-OF integral of Hahn polynomials. Then, we use a spectral collocation technique combined with the associated operational matrices of V-OF integral for solving weakly singular fractional integro-differential equations.

Material and methods

In this scheme, the operational matrix of fractional integration of Hahn polynomials is calculated. This method converts the weakly singular fractional integro-differential equations into an algebraic system which can be solved by a technique of linear algebra.

Results and discussion

In this paper, some numerical examples are provided to show the accuracy and efficiency of the presented method. By using a small number of Hahn polynomials, significant results are achieved which are compared to other methods. A comparison to the numerical solutions by CAS and Haar wavelets and Adomain decomposition method, shows that this technique is accurate enough to be known as a powerful device.

Conclusion

The following results are obtained from this research.

- The operational matrix of fractional integration of Hahn polynomials is presented for the first time.
- The main advantage of approximating a continuous function by Hahn polynomials is that they have a spectral accuracy at interval [0,N], where N is the number of bases.
- Furthermore, for estimating the coefficients of the expansion of approximate solution, we only have to compute a summation which is calculated exactly.
- Using Hahn polynomials, the numerical results achieved only by a small number of bases, are accurate in a larger interval and significant results are achieved.

Keywords: Weakly Singular Fractional Integro-Differential Equations, Hahn Polynomials, Operational Matrix, Spectral method.

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