Modules with Copure Intersection Property

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Extended Abstract

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Introduction

Throughout this paper, R will denote a commutative ring with identity and $\mathbb{Z}$ will denote the ring of integers.

Let M be an R-module. A submodule N of M is said to be *pure* if $IN = N \cap IM$ for every ideal I of R. M has *the copure sum property* if the sum of any two copure submodules is again copure. M is said to be a *comultiplication module* if for every submodule N of M there exists an ideal I of R such that $N = (0:_M I)$. M satisfies the *double annihilator conditions* if for each ideal I of R, we have $I = Ann_R((0:_M I))$. M is said to be a *strong comultiplication module* if M is a comultiplication R-module which satisfies the double annihilator conditions. A submodule N of M is called *fully invariant* if for every endomorphism $f: M \to M$, $f(N) \subseteq N$.

In [5], H. Ansari-Toroghy and F. Farshadifar introduced the dual notion of pure submodules (that is copure submodules) and investigated the first properties of this class of modules. A submodule N of M is said to be *copure* if $(N :_M I) = N + (0:_M I)$ for every ideal I of R.

Material and methods

We say that an R-module M has *the copure* intersection *property* if the intersection of any two copure submodules is again copure. In this paper, we investigate the modules with the copure intersection property and obtain some related results.

Conclusion

The following conclusions were drawn from this research.

- Every distributive R-module has the copure intersection property.
- Every strong comultiplication R-module has the copure intersection property.
- An R-module M has the copure intersection property if and only if for each ideal I of R and copure submodules N, K of M we have

$$N \cap K + (0:_M I) = (N + (0:_M I)) \cap (K + (0:_M I)).$$

- If R is a PID, then an R-module M has the copure intersection property if and only if M has the copure sum property.
- Let $M = \oplus_{i \in I} M_i$, where M_i is a submodule of M. If M has the copure intersection property, then each M_i has the has the copure intersection property. The converse is true if each copure submodule of M is fully invariant.

Keywords: Pure submodule, copure submodule, copure intersection property.

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