Solution of Some Boundary Value Problems Including Nonlinear Fractional Differential Equations

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Extended Abstract

Introduction

In this paper, by mixed monotone operators and their fixed points, we investigate the existence and uniqueness of positive solution for the following boundary value problems via nonlinear fractional differential equations.

$$D_{0+}^\alpha u(s,t) + f\big(s,t,u(s,t)\big) = 0, \tag{1}$$
$$(0 < \epsilon < T, \ \ T \geq 1, \quad t \in [\epsilon,T], \ \ 0 < \alpha < 1, \ \ s \in [a,b])$$
$$u(s,\eta) = u(s,T), (s,\eta) \in [a,b] \times (\epsilon,T), \qquad a,b \in (0,\infty), \qquad a < b$$

and

$$D_{0+}^\alpha u(t) = f\big(t,u(t)\big), \quad t \in (0,1) \ \ 3 < \alpha \leq 4, \tag{2}$$
$$u(0) = u'(0) = u(1) = u'(1) = 0,$$

where D^υ is the Riemann–Liouville derivative.

In recent years, fractional differential equations have been studied by many mathematicians both theoretically and practically, for example, in physics, mechanic, chemistry, engineering, biology, economy, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics and etc.

First time, mixed monotone operators have been introduced by Guo and Lakshmikantham in 1987. Next, many authors studied them in Banach spaces and obtained some results not only in theory fields but also wide applications in chemistry, engineering, biology, technology and other fields.

In 2009, Xu et. al studied the properties of Green's function for the nonlinear fractional differential equation boundary value problem (2). Here, the existence and uniqueness of its positive solutions are obtained by using the properties of cone and fixed point theorems for mixed monotone operators. As an application, we utilize the obtained results to study the existence and uniqueness of positive solution for nonlinear fractional differential equation boundary value problems.

Material and methods

The content of this paper is organized as follows. First, we present some definitions, lemmas and basic results that will be used in the proofs of our theorems. Then, we consider the existence and uniqueness of positive solution for the operator equation, also we utilize the results obtained to study the existence and uniqueness of positive solution for nonlinear fractional differential equation boundary value problems.

Results and discussion

In this work, we study the existence and uniqueness of positive solutions for nonlinear fractional differential equation boundary-value problems. Our results guarantee the existence of a unique positive solution, and can be applied for constructing an iterative scheme for obtaining the solution.

Conclusion

The following conclusions were drawn from this research.

Using the properties of cones and the fixed point theorem for mixed monotone operator, the existence and uniqueness of the positive solution are obtained. Our research methods are different from those in the related literature. As an application, we utilize the obtained results to study the existence and uniqueness of positive solution for nonlinear fractional differential equation boundary value problems. Our results extend and improve the related conclusions in the literature.

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