Limit Average Shadowing and Dominated Splitting

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Extended Abstract

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Introduction

The influence of persistence behavior of a dynamical system on tangent bundle of a manifold is always a challenge in dynamical systems. Persistence properties have been studied on whole manifold or on some pieces with independent dynamics. Since shadowing property has an important role in the qualitative theory of dynamical systems, by focusing on various shadowing properties, such as usual shadowing, inverse shadowing, limit shadowing, many interesting results have been obtained. The notion of limit shadowing property introduced by S. Pilyugin who obtained its relation to other various shadowing. Blank introduced the notion of average-shadowing property. It is known that every Axiom A diffeomorphism restricted to a basic set has the average shadowing property. K. Sakai proved that the C^1-interior of the set of all diffeomorphisms satisfying the average-shadowing property is characterized as the set of all Anosov diffeomorphisms.

Asymptotic average shadowing (AASP) defined by R. Gu for continuous maps, combines to the limit shadowing property with the average shadowing property. Here we modify the notion (AASP) and define the limit average shadowing for diffeomorphisms (LASP). R. Gu presented some basic properties of the limit average shadowing for continuous maps. He proved that if a continuous map has the limit average shadowing on a compact metric space X, then X is chain transitive and that L-hyperbolic homeomorphisms with limit average shadowing are topologically transitive. M. Kulczycki *et.al.*, found some relations between LASP and the other notion of topological dynamics. They proved that a surjective map with specification property has the LASP. Also, they found the relation between LASP and shadowing property. They also have been shown that an expansive continuous map with shadowing property is LASP if and only if it is mixing. This paper follows the ideas of R. Gu and M. Kulczycki *et.al.* Here we define LASP for diffeomorphism with a slight modification of the continuous case. We give an example which shows that shadowing property and LASP are not equivalent. Also, we introduce the notion of C^1- stably limit average shadowing for a closed f-invariant subset Λ of M, and show that if Λ is C^1-stably limit average shadowing and the minimal points of Λ are dense there, then Λ admits a dominated splitting.

Statement of the results

In this paper we give a system which has the limit average shadowing, but not the shadowing property. Also, one can give examples which have the shadowing but not the limit average shadowing property. Thus the limit average shadowing property does not imply the shadowing property. In fact, we can give a class of diffeomorphisms which have LASP, but not the shadowing property. In fact the following proposition gives a large class of diffeomorphisms satisfying the limit average shadowing.

Proposition A. *Let* Λ *be a locally maximal* f*-invariant set. If* Λ *is the specific set for* f *then* Λ *is limit average shadowable.*

The main purpose of the paper is to characterize the closed f-invariant set via limit average shadowing property in C^1-open condition. So, we consider the notion of limit average shadowing property in geometric differential dynamical systems. First we show that if f has the limit average shadowing property on a closed f-invariant set Λ then Λ is chain transitive. By using chain transitivity and limit average shadowing property we can prove that Λ is transitive.

Proposition B. If Λ is C^1-stably limit average shadowable, then there is a neighborhood $U(f)$ of f and a neighborhood U of Λ such that $\Lambda_g$contains neither almost sinks nor almost sources for any $g \in U(f)$.

Since we have proved that if f has the limit average shadowing property on a closed f-invariant set Λ and minimal points of f are dense then Λ is transitive. It is essentially proved that under assumptions and conclusions of the Proposition B, Λ admits a dominated splitting. Thus we get the main result of this paper.

Theorem C. *Let* Λ *be a closed f-invariant set whose minimal points are dense there. If* Λ *is* C^1*-stably limit average shadowing then* Λ *admits a dominated splitting.*

Keywords: Dominated splitting, Average shadowing, Limit shadowing, Asymptotic average shadowing, specification, Chain transitive, transitive, Mixing.

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