The Existence and Uniqueness of the Solution of a Nonlinear Partial Differential Equation in the Continuous Functions of Holder and Sobolev Spaces

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Introduction

Complex analysis is a comparatively active branch in mathematics which has grown significantly. A deep look at the implications of continuity, derivative and integral in complex analysis and their relation with partial differential equations determines the importance of establishing the relation between complex analysis and the theory of partial differential equations.

There are three aspects of the aims of complex analysis:

First it is possible to interpret the peculiarities of holomorphic functions as properties of solutions of specials systems of partial differential equations. Secondly, complex analysis becomes applicable to general classes of differential equations, not only to special ones. And thirdly, the new general complex analysis is able to construct solutions and to describe the properties of given solutions with the help of solutions of corresponding problems for holomorphic functions. The third aspect is significant, since in the case of nonlinear equations it essentially means their reduction to linear problems. In addition this third aspect shows that the general complex analysis is able to make use of results of classical function theory and of such results which originally have not been connected with partial differential equations.

One of the main goals of complex analysis is its systematic application in the branch of the theory of differential equations. By using the contraction function, the Gauss-Oatrogradski integral formula and the Banach fixed point theorem of the complex space, the classical and interesting method of existence and uniqueness of the solution of differential equations with nonlinear partial derivatives are smooth, which shows the importance of complex analysis in the branch of the theory of differential equations. In recent years, complex analysis and their relationship with partial differential equations have attracted a number of researchers.

In this paper, we determine the existence and uniqueness of a solution of a nonlinear partial differential equation in the form

where

$$F(z,\bar{z},w,p,q)=0,$$

$$z = x + iy, \quad w = w(z), \quad p = \frac{\partial w}{\partial z} = \frac{1}{2} \left(\frac{\partial w}{\partial x} - i \frac{\partial w}{\partial y} \right), \quad q = \frac{\partial w}{\partial \overline{z}} = \frac{1}{2} \left(\frac{\partial w}{\partial x} + i \frac{\partial w}{\partial y} \right).$$

Material and methods

Firstly, we transform the nonlinear partial differential equation in a complex space to a linear differential equation of the first order and show the existence of the solution. Then, we consider

the equivalence of the solution of the nonlinear partial differential equation with a singular integral equation system by using weak and strong singular integral operators and the property of the holomorphic function. Also, we consider the solution of the nonlinear partial differential equation on a bounded domain with finite area in the Holder space and Sobolev space.

Results and discussion

We obtain the uniqueness of the solution of the nonlinear partial differential equation by applying the Lipshitz condition in the Holder space and Sobolev space based on the contraction function and the Banach fixed point theorem.

Conclusion

In this paper, we discuss on the existence and uniqueness of the solution of the nonlinear partial differential equation in the continuous functions of Holder and Sobolev spaces in general case in complex space. We deduce the proposed method can be extended in other spaces.

Keywords: Nonlinear partial differential equation, Continuous functions of Holder, Sobolev space, Banach fixed point theorem.

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