Differential Operators and Differential Calculus on $\delta-$Hom-Jordan-Lie Superalgebras

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Extended Abstract

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Introduction

Hom-algebraic structures appeared first as a generalization of Lie algebras in [1,3], where the authors studied q-deformations of Witt and Virasoro algebras. A general study and construction of Hom-Lie algebras were considered in [7, 8]. Since then, other interesting Hom-type algebraic structures of many classical structures were studied Hom-associative algebras, Hom-Lie admissible algebras and Hom-Jordan algebras. Hom-algebraic structures were extended to Hom-Lie superalgebras in [2].

As a generalization of Lie superalgebras and Jordan Lie algebras, the notion of δ-Jordan Lie superalgebra was introduced in [6, 12] which is intimately related to both Jordan-super and atiassociative algebras. The case of δ=1 yields the Lie superalgebra, and we call the other case of δ=1 a Jordan Lie superalgebra, because it turns out to be a Jordan superalgebra. It is often convenient to consider both cases of δ= 1, and call δ-Jordan Lie superalgebras. The motivations to characterize Hom-Lie structurers are related to physics and to deformations of Lie algebras, in particular Lie algebras of vector fields. Hom-Lie superalgebras are a generalization of Hom-Lie algebras, where the classical super Jacobi identity is twisted by a linear map. If the skew-super symmetric bracket of a Hom-Lie superalgebra is replaced by δ-Jordan-super symmetric, it is called a δ-Jordan-Hom-Lie superalgebra (see [11]).

There are several notions of differential operators and differential calculus on non-associative algebras (see [4, 5]). A comprehensive definition of differential operators on non-associative algebras fails to be formulated. But many authors was studied a notion of differential operators and differential calculus on Lie algebras and Hom-Lie algebras [9, 10]. According to various applications in both mathematics and physics, we will investigate a notion of differential operators and differential calculus on multiplicative δ-Jordan-Hom-Lie superalgebras.

Material and methods

A key point is that the multiplications on multiplicative δ-Jordan-Hom-Lie superalgebras are their derivations. Therefore, definition of differential operators on a multiplicative δ-Jordan-Hom-Lie superalgebra must treat the derivations of this algebra as a first-order differential operators too. By our considerations, we will define higher order differential operators as composition of the first-order differential operators on a multiplicative δ-Jordan-Hom-Lie superalgebra. We also consider a geometric aspect to the concept of differential calculus on multiplicative δ-Jordan-Hom-Lie superalgebra by using the cohomology theory for this algebra.

Results and discussion

The theory of differential operators on associative algebras is not extended to the non-associative algebras in a straightforward way. But, we provide a notion of differential operators of any order on multiplicative δ-Jordan-Hom-Lie superalgebras and their modules. We also study some property of differential operators on multiplicative δ-Jordan-Hom-Lie superalgebras, for examples, the brackets and composition of two differential operators of higher order on these algebras. Finally, by using theory of cohomology for multiplicative δ-Jordan-Hom-Lie superalgebras, we investigate a notion of differential calculus on these algebras. In other words, for a multiplicative δ-Jordan-Hom- Lie superalgebra L with center Z(L) and Der(L), the derivation of L, we consider the cochain complex of L as Der(L)-module its subcomplex of Z(L)-multilinear morphism is said to be a differential calculus based on derivation of L. Next, we compute the differential calculus based on derivation of Hom-Lie super algebra osp(1, 2).

Conclusion

The following conclusions were drawn from this research.

- Definition of the differential operators of any order on multiplicative δ-Jordan-Hom-Lie superalgebras and prove several properties of it.
- Definition of the differential operators of any order on δ-modul of multiplicative δ-Jordan-Hom-Lie superalgebras and state some properties of it.
- The study of differential calculus based on derivation of a multiplicative δ-Jordan-Hom-Lie superalgebra.
- Compute the differential calculus based on derivation of Hom-Lie superalgebra osp (1, 2).

:**Keywords**Hom-Lie algebras, Hom-Lie superalgebras, Derivation and cohomology on Hom-Lie superalgebras

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