Application of the Modulus of Continuity in Characterizing Geodesics

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Extended Abstract

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Introduction

This paper concerns an application of the modulus of continuity in characterizing geodesics. The modulus of continuity of a continuous function between metric spaces is a two variable function which assigns to each point and to each positive epsilon the greatest positive delta that satisfies the definition of continuity at that point. It is shown that if f is a function of the real numbers into a metric space, then linearity of its modulus of continuity implies that f is locally geodesic, that is, it locally preserves metric.

Geodesics appear in several branches of mathematics as well as gravitational physics and general relativity. In mathematics, topics such as Riemannian geometry make extensive use of geodesics. In facts geodesics represents the shortest paths. So for example a mass that moves in the vacuum under the gravity passes through a geodesic.

Material and Methods

In Section 2 we first deal with some essential properties of the modulus of continuity. Then we mainly focus on continuous functions from the real numbers to metric spaces whose modulus of continuity is linear. In two theorems we characterize such functions and show that they are in fact locally geodesic.

Results and Discussion

The main result demonstrates that a continuous function with linear modulus of continuity falls into three classes: the class of functions that preserves metric with a coefficient, the class of functions that preserves metric on two rays whose intersection is a singleton, and the class of functions that preserves metric on two rays whose intersection is a closed interval of positive length. We then turn to locally convex metric spaces and show that the third class does not appear in such spaces. Finally, we apply our results to the case where both domain and range of function is the real line.

Conclusion

The following conclusions has been drawn in this paper:

- Essential properties of the modulus of continuity.
- Characterizing continuous functions of the real line into metric spaces whose modulus of continuity is linear.
- Dealing with continuous functions of the real line into locally convex metric spaces whose modulus of continuity is linear.

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