Characterization of Pseudo n-Jordan Homomorphisms Between Unital Algebras

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Extended Abstract

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Introduction

Let A and B be complex Banach algebras and $f: A \longrightarrow B$ be a linear map. Then f is called an n-homomorphism if for all $a_1, a_2, \dots, a_n \in A$,

$$f(a_1 a_2 \dots a_n) = f(a_1) f(a_2) \dots f(a_n),$$

and it is called an n-Jordan homomorphism if $f(a^n) = f(a)^n$, for all $a \in A$.
Clearly, every n-homomorphism is an n-Jordan homomorphism, but in general the converse is false. There are some examples of n-Jordan homomorphism which are not n-homomorphism. It is shown that every n-Jordan homomorphism between two commutative Banach algebras is an n-homomorphism for $n \in \{2,3,4\}$ and this result extended to the case $n \in \mathbb{N}$.

Zelazko proved that each Jordan homomorphism from Banach algebra A into a semisimple commutative Banach algebra B is a homomorphism. This result was proved by the first author for 3-Jordan homomorphism with the additional hypothesis that the Banach algebra A is unital, and it is extended by An for all $n \in \mathbb{N}$. Ebadian, Jabari and Kanzi introduced the new notation of n-Jordan homomorphism. Let A and B be algebras, and let B be a right A-module. Then a linear map $f: A \longrightarrow B$ is said to be pseudo n-Jordan homomorphism if there exists an element $w \in A$ such that $f(a^n w) = f(a)^n . \mathrm{w}$, for all $a \in A$. The element w is called Jordan coefficient of f. Obviously, every n-Jordan homomorphism from unital Banach algebra A into B that is unitary Banach A-module is a pseudo n-Jordan homomorphism.

Material and methods

Suppose that A and B are Banach algebras and B is a right A-module. It is shown under certain conditions that every pseudo n-Jordan homomorphism $f: A \longrightarrow B$ is a pseudo $(n+1)$-Jordan homomorphism. We first prove the converse of this Theorem based on the property of the Vandermonde matrix, and then we obtain some results concerning the relation between pseudo n-Jordan homomorphism and n-homomorphism by using the definition of separating point of Banach right A-module B.

Results and discussion

Under special hypotheses, we show that, every pseudo $(n+1)$-Jordan homomorphism f from Banach algebras A into Banach right A-module B is a pseudo n-Jordan homomorphism and every pseudo Jordan homomorphism $f: A \longrightarrow B$ is an (pseudo) n-Jordan homomorphism. The automatic continuity of pseudo n-Jordan homomorphism is also investigated.

Conclusion

The following conclusions were drawn from this research.

- Every unital pseudo $(n+1)$-Jordan homomorphism $f: A \longrightarrow B$ with a Jordan coefficient w is a pseudo n-Jordan homomorphism.
- Every unital pseudo Jordan homomorphism $f: A \longrightarrow B$ with a Jordan coefficient w such that w is a right separating point of B is an n-Jordan (pseudo n-Jordan) homomorphism.
- Every unital pseudo n-Jordan homomorphism $f: A \longrightarrow B$ with a Jordan coefficient w such that w is a right separating point of B is continuous either B is a semisimple and commutative, or B is semisimple and f is surjective.

Keywords: n-homomorphism, n-Jordan homomorphism, Pseudo n-Jordan homomorphism.

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