Numerical Solution of Fractional Black-Scholes Equation by Using Radial Basis Function (RBF) Approximation Method

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Extended Abstract

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Introduction

Fractional Differential Calculus (FDC) began in the 17th century and its initial discussions were related to the works of Leibniz, Lagrange, Abel and others. In recent decades, the fractional differential equations have been considered in different fields such as fluid flow, electromagnetics, engineering, economics and finance. In the early 1970s, Black and Scholes introduced their famous model for pricing option. This model is one of the most popular models in the financial market and plays an important role in determining the price of a high-risk asset in financial modeling field. In this equation, researchers seek to obtain the option value by numerical or analytical methods or to extract new pricing models that reflect the real financial market. The Black-Scholes equation is based on some assumptions which caused constraints in the market. Some advanced models such as jump-diffusion, stochastic interest rate, and stochastic volatility models have been proposed to remove these constraints. Wang and Meng (2010) show that the distribution of stock returns have long-range dependence property which is not consistent with the classical Black -Scholes equation assumptions. This led us to use the fractional modeling in this paper which is derived by applying the fractional specifications of stock market suggesting by Mandelbrot (1963). So, in this paper, we reach to the fractional Black-Scholes model by replacing fractional Brownian motion instead of standard Brownian motion in the classical Black-Scholes equation. The fractional Black-Scholes equation gives better solutions than classical Black-Scholes model to our data which their distribution of stock price depend on long-range. So, in this paper, we solve the fractional Black- Scholes equation and use the combination of radial basis functions and finite difference methods to solve the fractional Black-Scholes equation. This method is flexible because it does not depend on the position of points, and in comparison with other methods, it has a short run time in high dimensions.

Material and methods

In this paper, we reach to the fractional Black-Scholes equation by using the fractional underlying asset that follows the fractional Brownian motion. The model represents the price behavior of an European option. It is based on the stochastic behavior of underlying asset which is priced by fractional models. We also apply the radial basis function method to solve this model. In this method, we do not necessarily need to have points with equal distances and the convergence rate can be exponential. Therefore, this method can provide more acceptable

solutions than the other numerical methods. It should be noted that the fractional derivative of the pricing function is approximated by the Caputo fractional derivative. The stability of the proposed method has also been studied.

Results and discussion

In this paper, we provide the numerical results for the fractional Black-Scholes equation on real data of the coin option by radial basis function method. We receive the data of working days from 95/10/01 to 96/01/06 from Tehran Stock Exchange website by Excel software and uploaded them to MATLAB software format. The parameters required by the model are also estimated by using data. We obtain option price for different α by using these parameters and present the results in figures. Figures show that if the price of the underlying asset is lower than the exercise price, the option price decreases by the increasing of the fractional order (α), which means that if the holder chooses not to exercise the contract, he will incur less loss. Increasing α plays an important role in decreasing option price, if the option price decreases, the increasing α will make the purchaser incur less loss. On the other hand, when α increases, if the underlying asset price is above the strike price, the call option price increases, and the holder will make a gain by exercising it. We also predict the price of the coin for the next 5 days by different α. The results are presented in a table and are compared with real price of the market, which show that the method is efficient and the fractional Black-Scholes equation has better performance than the classical Black-Scholes equation.

Conclusion

The purpose of this paper is to model the fractional Black -Scholes equation and to solve it by radial basis function method. First we modeled the fractional Black-Scholes equation by using fractional underlying asset and then we solved this equation by radial basis function method. Finally, the efficiency of this method is shown by using the real data of the coin option. The numerical results present that the method is efficient, and the fractional Black-Scholes equation performs better than the classical Black- Scholes equation .The existence of α in the fractional Black -Scholes equation has drawn the prices closer to the real price.

Keywords: Fractional derivative, Fractional Black-Scholes equation, Radial basis function method.

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