Inverse Topology in BL-Algebras

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Received: 2017/11/05 Accepted: 2019/07/16

Extended Abstract

Paper pages (631-644)

Introduction

The first, the notion of BL-algebras have been introduced by Hajek, in order to provide an algebraic proof of the completeness theorem of “Basic logic”. It is arisen from the continuous triangular norms, familiar in the fuzzy logic framework. Next, Turunen published where BL-algebras were studied by deductive systems.

In fact, BL-algebras are particular residuated lattices. BL-algebras have been introduced by Hajek in order to investigate many valued logic by algebraic means.

His motivations for introducing BL-algebras were of two kinds.

The first one was providing an algebraic counterpart of a propositional logic, called Basic Logic, which embodies a fragment common to some of the most important many valued logics, namely Lukasiewicz Logic, Godel Logic and Product Logic. This Basic Logic (BL for short) is proposed as the most general many-valued logic with truth values in [0,1] and BL -algebras are the corresponding Lindenbaum-Tarski algebras.

The second one was to provide an algebraic mean for the study of continuous t-norm (triangular norms) on [0,1].

Then, Zariski topology in BL-algebras have been introduced by Leustean in 2003. He denoted the set of prime filters of A by $Spec(A)$.

For any $F \subseteq A$, let us denote the complement of $v_A(F)$ by $u_A(F)$. Hence $u_A(F) = \{P \in Spec(A): F \not\subset P\}$, $v_A(F) = \{P \in Spec(A): F \subseteq A\}$.

It follows that the family $\{u_A(F)\}_{F\subseteq A}$ is the family of open sets of the Zariski topology.

He show that this topology is T_0 and compact in $Spec(A)$ and for any $a \in A$, the family $\{u_A(a)\}_{a\in A}$ is a basis for the topology of $Spec(A)$.

Since the set $v_A(F)$ is closed set, for any $F \subseteq A$ in Zariski topology, while $V_A(F) = v_A(F) \cap Min(A)$ is open set in inverse topology, hence this topology is called inverse topology.

We note that F. Forouzesh et. al, introduced inverse topology in MV-algebras, we were also motivated to study the inverse topology in the BL-algebras.

In this paper, we introduce Inverse topology in a BL-algebra A and prove the set of all minimal prime filters of A, namely *Min(A)* with the Inverse topology is a compact space, Hausdorff, T_0 and T_1-Space. Then, we show that Zariski topology on *Min(A)* is finer than the Inverse topology on *Min(A)*.Then, we investigate what conditions may result in the equivalence of these two topologies. Finally, we define Min-extension in BL-algebra $A$and B show that the mapping $\psi: Min(A) \to Min(B)$ with respect to both the Zariski and the Inverse topology is continuous.

Conclusion

The following conclusions were drawn from this research.

- Inverse topology in a BL-algebra A is introduced and is proved the set of all minimal prime filters of A, namely *Min(A)* with the Inverse topology is a compact space, Hausdorff, T_0 and T_1-Space.
- It is shown that Zariski topology on *Min(A)* is finer than the Inverse topology on *Min(A)*.
- It is investigated, what conditions may result in the equivalence of these two topologies.
- Min-extension in BL-algebra A and B is defined and is shown that the mapping $\psi: Min(A) \to Min(B)$ with respect to both the Zariski and the Inverse topology is continuous.

Keywords: Minimal prime filter, Zariski topology, Inverse topology, Min-extension.

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