Some Tetravalent One-Regular Graphs of Special Order

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Extended Abstract

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Introduction

In this paper we consider undirected finite connected graphs without loops or multiple edges. For a graph X we use V(X), E(X), A(X) and Aut(X) to denote its vertex set, edge set, arc set and its full automorphism group, respectively. For u, $v \in$ V(X), {u,v} is the edge incident to u and v in X, and N(u) is the neighborhood of u in X, that is, the set of vertices adjacent to u in X. A graph X is said to be vertex-transitive and arc-transitive (or symmetric) if Aut(X) acts transitively on V(X) and A(X), respectively. In particular, if Aut(X) acts regularly on A(X), then X is said to be one-regular. Clearly, a one-regular graph is connected, and it is of valency 2 if and only if it is a cycle. In this sense the first non-trivial case is that of cubic graphs. The first example of a cubic one-regular graph was constructed by Frucht and later much subsequent work was done on this line. Also tetravalent one-regular graphs have also received considerable attention.

For example tetravalent one-regular graphs of prime order were constructed. Furthermore, the classification of tetravalent one-regular graphs of order $3p^2$, $4p^2$, $7p^2$ and 2pq are classified. Along this line the aim of this paper is to classify tetravalent one-regular graphs of order $11p^2$.

Material and methods

For proving the main result algebraic and topological Methods in Graph Theory are used. Also we use some results in permutation groups.

Result

The following result is the main result of this paper.

Therem. Let p be a prime. A tetravalent graph X of order $11p^2$ is 1-regular if and only if one of the following holds:

(i) $p \in \{2,3,5,7\}$,

(ii) X is a Cayley graph over $< x,y \mid x^p = y^{11p} = [x,y] = 1 >$, with connection set $\{y, y^{-1}, xy, x^{-1}y^{-1}\}$,

(iii) X is connected arc-transitive circulant graph with respect to every connection set S,

(iv) X is one of the graphs described in [1, Lemma 8.4].

Keywords: s-Transitive graphs, Symmetric graphs, Cayley graphs

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