A New Inverse Eigenvalue Problem for Jacobi Matrices and Corresponding Mass-Spring System

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Extended Abstract

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Introduction

Many problems in sciences and engineering can be studied by mathematical models. These models are classified as direct problems and inverse problems. In the structural vibrations, analysis and estimation of the behavior of system e.g. response of the system for an external force and natural frequencies from the known physical parameters is called a direct problem. Determination or estimation of the system physical parameters such as density, mass, stiffness and cross sectional area from the behavior of the system is called an inverse problem. A class of inverse problems which physical parameters determined from the spectral data (eigenvalues, eigenvectors, or both) is called inverse eigenvalue problem. There are many systems such as mass- spring system, vibrating Rods and Beams which are modeled as an eigenvalue problem. Free vibrations of a mass- spring system and discretization of a rod and Sturm-Liouville equations lead to Jacobi matrix eigenvalue problem. Inverse eigenvalue problem for Jacobi matrix is determination of entries using some spectral data. Different algorithms have been presented for constructing a Jacobi matrix. In this paper, we construct a Jacobi matrix and the corresponding mass-spring system using some new spectral data.

Material and methods

We try to construct a Jacobi matrix from two spectra and one extra data. For this purpose, using given spectral data, we find the required data of well-known Lancsoz method. Then applying Lancsoz method, we construct a positive definite Jacobi matrix. Finally, according to the relations between Jacobi matrix, mass and stiffness matrices, we obtain corresponding mass-spring system.

Results and discussion

Necessary and sufficient conditions on given spectral data for solvability of the inverse eigenvalue problem are presented.

We find two algorithms for constructing positive definite Jacobi matrix and the corresponding mass-spring system.

We solve some examples using the given algorithms. There is a good agreement between the spectral data of constructed matrix and initial given data.

Conclusion

The following results are obtained from this research.

- We find two algorithms for constructing a Jacobi matrix using two spectra and one extra data.
- It is observed that, for a set of spectral data, there might be exist more than one solution.

- It seems that, one may extend the method of this paper for matrix eigenvalue problem which arise in discretization of vibrating rod using finite element method.

Keywords: Inverse eigenvalue problem, Jacobi matrix, Spectral data, Mass-Spring system.

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