Numerical Solution Using Chebyshev Expansion of the Higher-Orders Linear Fredholm Integro-Differential-Difference Equations with Variable Coefficients

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Extended Abstract

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The main aim of this paper is to apply the Chebyshev polynomials for the solution of the linear Fredholm integro-differential-difference equation of high orders. It is usually difficult to analytically solve this equation. Our approach consists of reducing the problem to a set of linear equations by means of the matrix relations between the Chebyshev polynomials and their derivatives. The operational matrices of delay and derivative together with the Tau method are then utilized to evaluate the unknown coefficients of Chebyshev expansion of the solution. The convergence analysis is studied. Illustrative examples show the validity and applicability of the presented technique. Also, a comparison is made with existing results.

Introduction

The integro-difference equations arise in different applications such as biological, physical and engineering problems. In recent years, there has been a growing interest in the numerical treatment of the integro-differential-difference equations. Since the mentioned equations are usually difficult to solve analytically, numerical methods are required. Several numerical methods were used such as successive approximation method, Adomian decomposition method, the Taylor collocation method, Haar wavelet method, Legendre wavelets method, wavelet-Galerkin method, monotone iterative technique, Walsh series method, etc.

In this work, we develop a framework to obtain the numerical solution of the s-order linear Fredholm integro-differential-difference equation with variable coefficients.

$$\sum_{k=0}^{s} p_k(x)y^{(k)}(x)+\sum_{r=0}^{t} p_r^*(x)y^{(r)}(x-\tau)= f(x)+\int_{-1}^{1}k(x,t)y(t-\tau)dt, \quad \tau\geq 0,$$

under the mixed conditions

$$\sum_{k=0}^{s-1}(a_{ik}y^{(k)}(-1)+b_{ik}y^{(k)}(1)+c_{ik}y^{(k)}(0))=\mu_i, \qquad i=0,1,...,s-1,$$

where p_k, p_r^*, k and f are known continuous functions. Here, the real coefficients a_{ik}, b_{ik}, c_{ik} and μ_i are given constants.

Our approach consists of reducing the problem to a set of linear equations by expanding the solution y in terms of Chebyshev polynomials. The operational matrices of delay and derivative are given. These matrices together with the Tau method utilized to evaluate the unknown coefficients of expansion. The Tau method has been originally proposed by Lanczos for ordinary differential equations and extended by Ortiz. The method consists of expanding the required approximate solution as the elements of a complete set of orthogonal polynomials. Recently there have been several published works in the literature on the applications of the Tau method.

Conclusion

This paper deals with the solution of linear Fredholm integro-differential-difference equations of high order with variable coefficients. Our approach was based on the Chebyshev Tau method which reduces a linear Fredholm integro-differential-difference equation into a set of linear algebraic equations. Numerical results show that this approach can solve the problem effectively. The approach, with some modifications, can be employed to solve differential-difference equations and Fredholm integro-differential equations.

Keywords: Differential-difference equation, Fredholm integro-differential-difference equation, Tau method, Operational matrix, Chebyshev polynomials.

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