On Lie Symmetries of Hyperbolic Model Metric of SL(n, R) Geometry

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Extended Abstract

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Introduction

Symmetries of an equation are closely related to conservation laws. Noether's theorem provides a method for finding conservation laws of differential equations arising from a known Lagrangian and having a known Lie symmetry.

An algorithmic method to determine conservation laws for systems of Euler-Lagrange equations which their Noether symmetries are known is Noether theorem. This theorem relies on the availability of a Lagrangian and the corresponding Noether symmetries which leave invariant the action integral.

One can find the geodesic equations from the variation of the geodesic Lagrangian defined by the given metric and since the Noether symmetries are a subgroup of the Lie group of Lie symmetries of these equations, one should expect a relation of the Noether symmetries of this Lagrangian with the projective collineations of the metric or with its degenerates.

Recently Bokhari et al. have published many papers about the relation of the Noether symmetries and Lie point symmetrys and conservation laws of some special spacetimes.
Tsamparlis and Paliathanasis have calculated the Lie point symmetries and the Noether symmetries explicitly together with the corresponding linear and quadratic first integrals for the Schwarzschild spacetime and the Friedman Robertson Walker (FRW) spacetime. More than they succeeded in establishing a connection between the Lie symmetries of the geodesic equations in a Riemannian space with the collineations of the metric.

Results and discussion

In this paper, Lie and Noether symmetries and conservation laws for the hyperbolic model metric of SL(2, R) geometry in a Riemannian space are obtained. Then the point symmetries of the one parameter Lie groups of transformations that leave invariant the action integral corresponding to the Lagrangian that means Noether symmetries are found and then the conservation laws associated to the system of geodesic equations are calculated via Noether's theorem.

Conclusion

The following conclusions were drawn from this research.

- The Lie point symmetry algebra of the hyperbolic model metric SL(2, R) geometry has six dimensions which would include the Noether symmetries, have an additional basis element $s\frac{\partial}{\partial s}$ which is dilation in the arclength.

- The Lie point symmetry $X_5 = \frac{\partial}{\partial \theta}$ commutes with all other symmetries. As a more discussion in Lie algebra analysis, one may find the Lie algebra of Lie point symmetry corresponding to the metric is not semisimple or not solvable, because it has degenerated Killing form .
- There are five conserved flows corresponding to the Noether symmetries of given metric.

Keywords: Conservation laws ,Lie point symmetry ,LagrangianRiemannian space, Noether theorem, Hyperbolic model metric.

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