E-Bayesian Estimations of Reliability and Hazard Rate in the Generalized Inverted Exponential Distribution Based on Type II Censoring

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Extended Abstract

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Introduction

Lifetime tests and Estimation of survival functions such as reliability and hazard rate (failure rate) is necessary in many applications. These functions have a vast variety of applications in statistics and many branches of engineering. Reliability engineering deals with the estimation and management of high levels of lifetime and risks of failure. Hence, the estimation of reliability and hazard rate are important in different sciences. To this aim, different methods of estimation have been employed. Each method suffers from its own problems such as complexity of calculations, high risk, low precision and so on. The most famous old methods of estimating are maximum likelihood and Bayes. Each method exploits various tools. Now, this study employed a new method, named E-Bayesian (Expected Bayesian), for estimating the unknown parameter, reliability (Series and parallel systems) and hazard rate functions of the Generalized Inverted Exponential Distribution, which is one of the most noticeable distributions in lifetime studies. Because, in many situations, this distribution has a better level of fitting than the other similar distributions such as the gamma, weibull, generalized exponential and inverted exponential distributions and due to its practicality, this distribution can be used for many applications, including engineering and technical sciences such as accelerated life testing, sea currents, wind speeds, queuing theory, horse racing and etc. Also, the estimates are derived based on a conjugate prior for the unknown parameter and a squared error loss function. Furthermore, E-Bayesian estimations are obtained based on different priors of the hyperparameters to investigate the influence of different prior distributions on these estimations. On the other side, statistical methods dealing with censored data have a long history in the field of survival analysis and life testing. Related to this topic, the results of this paper are calculated using type-II censored data which is more economical than the type-I censoring. Besides, the asymptotic behaviors of E-Bayesian estimations and relations among them have been investigated. Finally, a comparison among the maximum likelihood, Bayes and E-Bayesian estimations in different sample sizes are made, using a real data and Monte Carlo simulation.

Material and methods

In this scheme, we first suppose the Generalized Inverted Exponential Distribution and its unknown parameter, reliability and hazard rate functions. Then the estimates of functions of interest are derived based on type-II censored samples of this distribution and under conjugate prior for the unknown parameter and squared error loss function. The most of integrals of this paper cannot be solved analytically in a simple closed form. In other words, in most cases this is

quite difficult if not impossible. To do this, attempts are made to use numerical computations by R packages and Monte Carlo simulation.

Results and discussion

In this paper, the maximum likelihood, Bayes and E-Bayesian techniques are used for estimating the parameter, reliability and hazard functions of the Generalized Inverted Exponential Distribution based on type-II censoring. A real data and Monte Carlo simulation are used for computing and comparisons of these estimate methods. The reported results demonstrate that the new presented method is more efficient than previous methods and is also easy to operate. Also, the numerical results reported in the tables indicate that the asymptotic behaviors of three E-Bayesian estimations in any parameter, under different priors of the hyperparameters are the same. In addition, overwhelming majority of E-Bayesian estimations are better than previous estimations in the sense of having smaller estimated risks of the estimates and also exploiting lower computational complexity. Furthermore, the estimated risks of the estimates decrease as sample size increases.

Conclusion

The following conclusions were drawn from this research.

- The E-Bayesian estimates of different priors are more nearly as the sample size increases. By different prior distributions of hyperparameters, the E-Bayesian estimations are robust and satisfy the theorems.
- The estimated risks of estimates decrease as the sample size increases.
- The estimated risks of the E-Bayesian estimates based on the SELF, were less than the estimated risks of their corresponding MLE and Bayes estimates.
- Generally, this paper shows that the E-Bayesian estimation is more efficient than other old methods and is easy to operate.

Keywords: E-Bayesian estimation, Type-II censoring, Reliability, Hazard Rate, Monte Carlo simulation

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