A Numerical Approach Based on Operational Matrix for Solving Fractional Delay Differential Equations

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Extended Abstract

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Introduction

Fractional calculus has been used to model physical and chemical processes that are found to be best described by fractional differential equations. Recently, fractional calculus has attracted much attention since it plays an important role in many fields of science and engineering. Fractional delay differential equations (FDDEs) are a class of fractional differential equations that the rate of change of unknown function depends not only on the values of unknown function for the same time value but also on previous time values. The solution of delay differential equations not only requires information of current state, but also requires some information about the previous state. FDDEs have received considerable recent attention and been proven to model many real life problems. For most of fractional order delay differential equations, exact solutions are not known. Therefore different numerical methods have been developed and applied for providing approximate solutions. The objective of this paper is to define the new fractional-order piecewise functions based on the Taylor polynomials for solving the FDDEs. This method is accurate and easy to implement in solving FDDEs.

Material and methods

In this paper, first we construct piecewise functions based on the fractional-order Taylor functions. Then, we calculate the fractional integral operational matrix for the fractional Taylor piecewise functions. This matrix and collocation method are utilized to reduce FDDEs to a system of algebraic equations which can be solved via a suitable numerical method.

Results and discussion

We apply mentioned paper for solving some test problems to highlight the significant features of our technique. Also, we compare our numerical results with multiquadric approximation scheme. The reported results demonstrate that there is a good agreement between approximate solution and exact solution. We plot the numerical solutions obtained by the presented method for various values of α with the exact solution. It is obvious from these Figures that, as α is close to integer value, numerical solutions converge to the exact solution.

Conclusion

The following conclusions were drawn from this paper.

Fractional-order Taylor piecewise functions have three degrees of freedom (m, n, α) but Taylor polynomials have one degree of freedom (m).

- Instead of converting fractional-order Taylor piecewise functions into other functions, we have obtained the fractional-order Taylor piecewise functions operational matrix directly.
- The main characteristic of this method is that it reduces under study problem to a system of

algebraic equations which can be easily solved by an iterative method.

Keywords: Fractional delay differential equations, Collocation method, Fractional-Taylor function, Caputo derivative, Operational matrix.

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