A Description of Commutant of Multiplication Operators on Bergman Spaces of the Polydisk

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Abstract

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An important problem in the operator theory of analytic function spaces, and in particular, in the operator theory of Bergman spaces is to describe the commutant of specific operators. Here, we mean by the commutant of a given operator T acting on the Bergman space $A^2(D)$, the algebra of all operators S that commute with T; that is, $TS = ST$.

It is well-known (see for instance, [2]) that if T is the operator of multiplication by z, then S commutes with T if and only if S equals M_ϕ where ϕ is a bounded analytic function on the open unit disk of the complex plane; here the multiplication operator M_ϕ is defined in the following way:

$$M_\phi(f) = \phi f.$$

The problem is much more difficult if we replace z by z^2 for instance. This problem was solved by Kehe Zhu in [3] in 2000. Zhu proved that T belongs to the commutant of $f \mapsto z^2 f$ if and only if there are two bounded analytic functions ϕ and ψ in the open unit disk such that

$$Tf = \frac{\phi f_o}{z} + \psi f_e,$$

where $f = f_e + f_o$ is the decomposition of f to an even and an odd function. This means that the commutant is the direct sum of two copies of the space of bounded analytic functions $H^\infty(D)$.

Following that, the author generalized this theorem to the case of weighted Berman spaces [1].

In this paper we intend to investigate the case of polydisk $D^2 = D \times D$. We fix one variable and consider the operator

$$f(z_1, z_2) \mapsto M(f) = z_1^2 f(z_1, z_2).$$

It is proved that T commutes with M if and only if there are two bounded analytic functions $h_0(z_1, z_2)$ and $h_1(z_1, z_2)$ in the polydisk D^2 such that

$$Tf = h_0 f_0 + \frac{h_1 f_1}{z_1}$$

where $f = f_0 + f_1$ is the even-odd decomposition of f in terms of the first variable, more precisely,

$$f_0(z_1, z_2) = \frac{f(z_1,z_2) + f(-z_1,z_2)}{2},$$

$$f_1(z_1, z_2) = \frac{f(z_1, z_2) - f(-z_1, z_2)}{2}.$$

The problem can easily be generalized to the case $D^n = D \times \cdots \times D$.

Keywords: Weighted Bergman space, Commutant of operator, Multiplication operator, Polydisk

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