Biflatness of Abstract Segal Algebras Based on Characters

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Abstract

In this paper, we investigate and study the notion of left ϕ-biflatness of abstract Segal algebras, where ϕ is a character on Banach algebra. Precisely, we give a necessary and sufficient condition for left ϕ-biflatness of abstract Segal algebras equipped with a left approximate identity. As an application, we show that if $S(G)$ is a Segal algebra on the locally group G and $\phi: L^1(G) \rightarrow \mathbb{C}$ is a character, then $S(G)$ is left $\phi|S(G)$-biflat if and only if G is amenable. Indeed, this is a generalization of [4, Theorem 3.4]. Moreover, we study the relationship between left ϕ-biflatness and inner ϕ-amenability and show that if the Banach algebra A is inner ϕ-amenable, then the notions of left ϕ-biflatness and left ϕ-amenability are equivalent.

Keywords and phrases: Abstract Segal algebra, left ϕ-biflat, left ϕ-amenable, inner ϕ-amenable.

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Extended Abstract

Introduction

The notion of biflatness of Banach algebras was defined by A. Ya. Helemskii to study the structure of Banach algebras via homology theory. It is well-known that the group algebra $L^1(G)$ is biflat if and only if G is amenable. It is also shown that a biflat Banach algebra with a bounded approximate identity is amenable. Biflatness and amenability of some concrete Banach algebras like semigroup algebras, Fourier algebras and measure algebras were studied in many papers. E. Kanuith et al. introduced a notion of amenability with respect to a non-zero multiplicative functional Φ for Banach algebras, called left Φ-amenability. This notion has been studied for large classes of Banach algebras such as the abstract Segal algebras, Fourier algebras, measure algebras. Motivated by these considerations, Sahami et al. introduced and studied a notion of biflatness related to a non-zero multiplicative functional, say left Φ-biflatness. It is worthwhile to mention that Essmaili et al. introduced and studied a condition on Banach algebra called the condition (W). It is easy to see that the condition (W) is coincide with left Φ-biflatness with a small modification. It is shown that the Lebesgue-Fourier algebra $LA(G)$ is left Φ-biflat if and only if G is amenable. Moreover, left Φ-biflatness of symmetric abstract Segal algebra has been studied in some papers. As a consequence for symmetric Segal algebra $S^1(G)$, it is proved that left Φ-biflatness is equivalent with amenability of G.

In this paper, we investigate and study the notion of left Φ-biflatness of abstract Segal algebras, where Φ is a character on Banach algebra. Precisely, we give a necessary and sufficient condition for left ϕ-biflatness of abstract Segal algebras equipped with a left approximate identity. As an application, we show that if $S(G)$ is a Segal algebra on the locally group G and $\Phi: L^1(G) \to \mathbb{C}$ is a character, then $S(G)$ is left $\Phi|S(G)$-biflat if and only if G is amenable. Indeed, this is a generalization of a well-known result. Moreover, we study the relationship between left Φ-biflatness and inner Φ-amenability and show that if the Banach algebra A is inner Φ-amenable, then the notions of left Φ-biflatness and left Φ-amenability are equivalent.

Main Results

We first study the relationship between left Φ-biflatness and left Φ-amenability for abstract Segal algebras and prove the following results.

Lemma 1: Suppose that A is a left Φ-biflat Banach algebra such that $A\ker\Phi$ is norm dense in $\ker\Phi$. Then A is left Φ-amenable.

Theorem 2: Let A be a Banach algebra, $\Phi \in \Delta(A)$ and B be an abstract Segal algebra with respect to A which possess a left approximate identity. Then the following statements are equivalent:

i) A is left Φ-biflat.
ii) B is left $\Phi|B$-biflat.
iii) B is left $\Phi|B$-amenable.
iv) A is left Φ-amenable.

As a consequence, we characterize left Φ-biflatness of Segal algebras as follows:

Corollary 3: Suppose that G is a locally compact group, $S(G)$ is a Segal algebra on G and $\Phi \in \Delta(L^1(G))$. Then the following are equivalent:

i) $L^1(G)$ is left Φ-biflat.
ii) $S(G)$ is left $\Phi|S(G)$-biflat.
iii) G is amenable.

Theorem 4: Let A be a Banach algebra with a left approximate identity and $\Phi \in \Delta(A)$. Suppose that B is an abstrect Segal algebra with respect to A. Then B is left $\Phi|B$-biflat if and only if B is left $\Phi|B$-amenable.

In the consequence, we investigate left Φ-biflatness of Φ-inner amenable Banach algebras. Indeed, for a Φ-inner amenable Banach algebras we prove that left Φ-biflatness and left Φ-amenability are equivalent.

Lemma 5: Let A be a Φ-inner amenable Banach algebra for $\Phi \in \Delta(A)$. Then A is left Φ-biflat if and only if A is left Φ-amenable.

Theorem 6: Let A be a Banach algebra, $\Phi \in \Delta(A)$ and A be Φ-inner amenable. Suppose that B is an abstract Segal algebra with respect to A. Then B is left $\Phi|B$-biflat if and only if B is left $\Phi|B$-amenable.

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